OPEN ACCESS
Indo-JC Indonesia Journal of Computing
ISSN 2460-9056
socj.telkomuniversity.ac.id/indojc

Ind. Journal on Computing
Vol. 5, Issue. 1, March 2020. pp. 15-22
doi:10.21108/indojc.2020.5.1.352

Polish Rod Displacement Simulation on Sucker Rod Pump Using the Piecewise-Linear Basis Method

I.G.M Bagus Nurseta Wijaya [1], A. Aditsania [2], P. H. Gunawan [3]
School of Computing, Universitas Telkom
Jalan Telekomunikasi No. 1 Terusan Buah Batu, Bandung 40257, Indonesia
[1] bagusnurseta@student.telkomuniversity.ac.id
[2] aaditsania@telkomuniversity.ac.id
[3] phgunawan@telkomuniversity.ac.id

Abstract

Sucker Rod Pump (SRP) is a heavy equipment used to pump oil(fluid) from the ground. The purpose of this study is to determine the numerical solution of displacement problem on the polish rod in the sucker rod pump. From previous studies displacement can be simulated and calculated by numerical solutions using the finite difference method. Here, the Piecewise-Linear Base method is used to simulate displacement on the polish rod. This method has higher accuracy than finite difference method. Moreover the numerical solution will be compared to the analytic simulation of the problem to confirm the robustness of the numerical method. From the simulation results, L1 error rates are obtained 2.00911 x 10^{-17} at $\Delta x = 0.2$, 3.30195 x 10^{-17} at $\Delta x = 0.05$ and 1.45114 x 10^{-16} at $\Delta x = 0.02$.

Keywords: Sucker rod pump, fluid, Rayleigh-Ritz, Piecewise-Linear Basis, polish rod, displacement, L1 error.

Abstrak

Sucker Rod Pump(SRP) adalah sebuah alat berat yang digunakan untuk memompa minyak (*fluida*) dari dalam tanah. Tujuan dari penelitian ini untuk menetukan solusi numerik dari permasalahan *displacement* pada batang polish rod di sucker rod pump. Dari penelitian sebelumnya regangan (*displacement*) dapat disimulasikan dan di hitung solusi numeriknya menggunakan metode beda hingga. Pada paper ini, metode Piecewise-Linear Basis digunakan untuk menyimulasikan regangan(*displacement*) pada batang polish rod. Metode ini memiliki tingkat akurasi yang lebih tinggi dari metode beda hingga. Selain itu, solusi numerik yang didapatkan, dibandingkan dengan simulasi analitik dari *problem* tersebut untuk melihat kemampuan metode yang diusulkan. Dari hasil simulasi didapat nilai *L1 error* sebesar 2.00911 x 10^{-17} at $\Delta x = 0.2$, 3.30195 x 10^{-17} at $\Delta x = 0.05$ and 1.45114 x 10^{-16} at $\Delta x = 0.02$.

Kata Kunci: Sucker rod pump, fluida, Rayleigh-Ritz, Piecewise-Linear Basis, polish rod, *displacement*, *L1 error*.

I. Introduction

SUCKER rod pump (SRP) which commonly called as beam pump in Indonesia, is a kind of pump that is widely used in the field for transporting heavy oil (fluid). At first, the drilling of oil can flow on its own because of the high pressure inside the well [1]. However, the well pressure decreases overtime, hence the oil could not flow to the surface by its own [1]. Sucker rod pump is the main component that has an important role in mining oil in wells. The sucker rod pump has 2 main parts, which are the above ground and underground part. Many researchers have conducted research or trials and numerical measurements of polish rod stretching on those sucker rod pump. Those numerical measurement are used to determine the displacement stretching that can be done by the polish rod in the underground part of the beam pump for transporting oil (fluid) [9].

Received on Jan 2020. Accepted on March 2020

Sucker rod pump is a tool, which works by transporting oil with up and down movement. The up and down movement resulted in the forming of displacement at sucker rod, precisely on the polish rod. Displacement on polish rod is modelled with wave equation with the following boundary and initial conditions:

$$\frac{\partial^2 \mu(x,t)}{\partial t^2} = C_w^2 \frac{\partial^2 \mu(x,t)}{\partial x^2} \tag{1}$$

$$\mu \mid_{t=0} = 0, \frac{\partial \mu}{\partial t} \mid_{t=0} = V \bullet \frac{y}{L}, \mu \mid_{x=0} = 0, \frac{\partial \mu}{\partial x} \mid_{x=L} = 0 \tag{2}$$

where C_w is a propagation speed, μ s the displacement position of the segment, V is the relative speed of the rod string. L is the total length of the string, y is the number of segment of the sucker rod. Moreover, Eqs.(1) - (2) defined displacement in the sucker rod on each segmen(x) and time(t). This model is similar with research conducted by Xinfu Liu et al., [9], where the wave equation is used to represent displacement of the polish rod.

The Eq.(1) is a wave equation, which is used to find the numeric solution of displacement and will be discussed in this research. Numerical measurement of (1) has been done using Finite Difference Method (FDM), however the analysis of this method is not elaborated in detail. In this research, Rayleigh Ritz Piecewise-Linear Basis method will be used to stimulate displacement in polish rod. This method is known as a high accurate method in second order spatial approximation. Therefore, the analytical solution of the wave equation will be used to investigate the robustness of numerical solution. The wave equation is a part of partial differential equation, hence the finite difference and Rayleigh Ritz methods can be used in this problem [11].

II. Literature Review

A. Sucker Rod Pump

Sucker rod pump is one of the oldest pump which is still used nowadays. This tool has been existed for 155 years [2]. This tool is one the most favorite tool used in mining several liters of oils (fluid) because of its simplicity. Sucker rod pump is a specifically designed pump, for transporting fluid from underground to the surface [2]. This pump is one of the favorite pump used to drill oil, it used basic component such as piston. The principle work of the sucker rod beam pump is using the drive motor to convert electrical energy into rotational motion [5]. General figure of sucker rod pump components can be seen in Fig.1.

The common way of sucker rod pump works is by converting rotational motion (prime mover) to up and down motion. After that, this pump has up and down motion which will be transported through walking beam and will be forwarded to the horsehead. The motion up and down will become up stroke and down stroke motion, which will move the pump plunger through the rod string [5].

This tool commonly have two parts, which is the above the surface and the underground parts [5]. The equipment above the surface is used to move the energy from prime mover to pumping unit, where it is being forwarded into the underground pump later. It also used to change the rotational motion into up and down motion through crank, pitman and walking beam. Meanwhile the gear reducer is used for decreasing the high rotation of the prime mover, in accordance with the stroke per minute of the pump [7]. The above surface parts consist of crank, prime mover, gear reducer, counter weight, pitman arm, walking beam, horsehead, bridle, polished rod, stuffing box, and brake.

In the underground sucker rod pump, there are 4 main components:

- Working barrel, which is a cylindrical tube where the plunger is moving up and down.
- Plunger, which is a long piston made of stainless steel and moves up and down (in accordance with pumping principal).
- Traveling valve, which is functioned to transporting the oil (fluid) from the bottom of the well to the tubing column up to the surface. This valve will open when the plunger moved down stroke, and will close when upstroke [7].
- The last component is standing valve, which is a ball-shaped valve, and located at the bottom part of the pump, and used to hold down the oil (fluid) so that it does not come out from the working barrel when down stroking [7], see in Fig.1 for more detail.

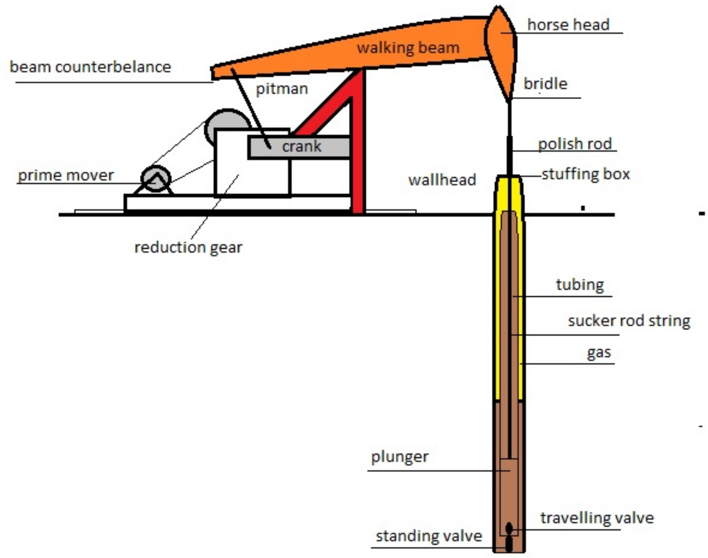

Fig. 1. The components of Sucker Rod Pump.

B. Rayleigh Ritz Method

Rayleigh Ritz method is used to find the estimate of Eigen value that cannot be solved analytically with ease [3] or using solving boundary value problem method, boundary value problem is changed with a pair of initial value problem. Rayleigh Ritz method find value basic Eigen approach from a matrix [8]. Ritz method in numerical analysis is method for changing continuous operator problems into discrete. This method is similar in application with method of variation into function space by changing the equation into weak formula. To describe Ritz method, consider a solution approach to the problem of linear two-point boundary values from beam-stress analysis. This boundary values problem can be explained with differential equation [4].

$$-\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right)+q(x)y=f(x), for 0\leq x\leq 1, \tag{3}$$

with boundary condition

$$y(0)=y(1)=0. \tag{4}$$

where $y(x)$ deflection of the beam length, with a variable cross-section length by $q(x)$. The deflection is caused by additional pressure $p(x)$ and $f(x)$ [4].

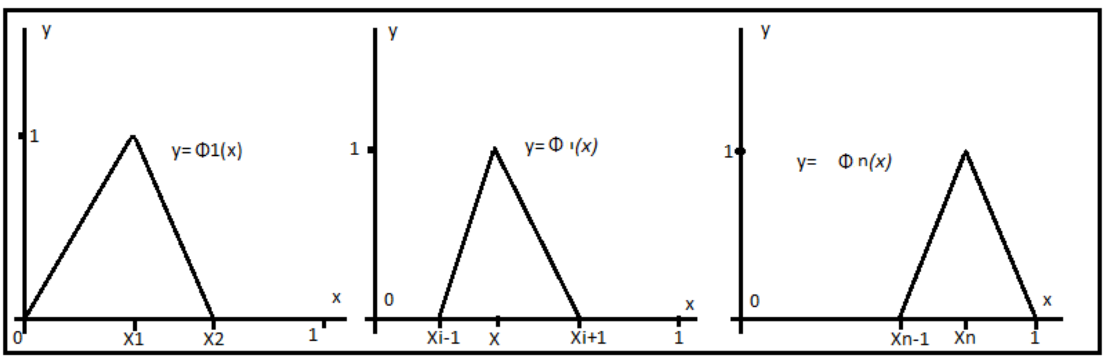

Fig. 2. The piecewise-linear configuration for describing Eq. (5).

Giving $\phi(x)$ is a function to approach approach of $y(x)$. Then in case $\phi(x)$ is linear function, then it can be written as $y(x) \approx \phi(x) = \sum_{i=1}^{n} c_i\phi_i(x)$. In $[0,1]$ partition, by giving a positive integer for n and defining $h_i = x_{i+1} - x_i$ for each $i = 0, 1, \cdots, n$, the basic function $\phi_1(x), \phi_2(x), ..., \phi_n(x)$ can be given as the following equation and described as in Fig. 2.

$$\phi_i(x) = x \begin{cases} 0, & \text{if } 0 \leq x \leq x_{i-1}, \\ \frac{1}{h_{i-1}}(x - x_{i-1}), & \text{if } x_{i-1} < x \leq x_i \\ \frac{1}{h_i}(x_{i+1} - x), & \text{if } x_i < x \leq x_{i+1} \\ 0, & \text{if} x_{i+1} < x \leq 1 \end{cases} \tag{5}$$

III. Methodology and Built System

A. Flowchart System

The purpose of this study is to determine the numerical solution of the displacement problem of the polish rod at sucker rod pump, by using Rayleigh Ritz Piecewise-Linear Basis method. The way this research works is by discretizing problem, then build a numerical and analytical calculation problem as a comparison of this method. The research Flowchart is described in Fig. 3:

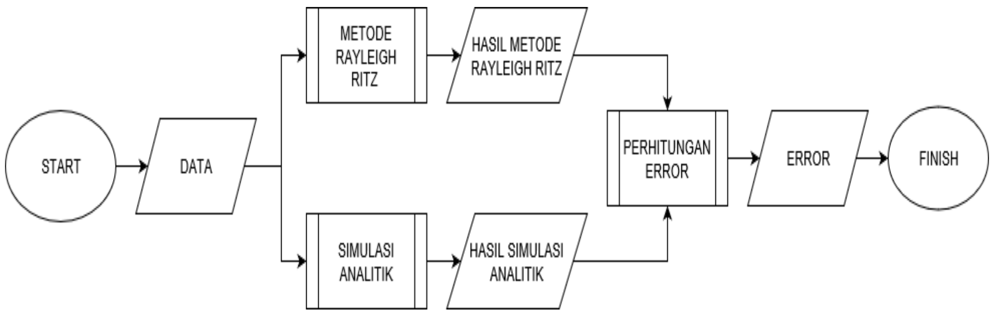

Fig. 3. The flow of this research.

B. Discrete model of Piecewise-Linear Basis

The Eq.(1) must be formed into Eq.(3), so that it can be decreased into discrete form of Rayleigh Ritz piecewise-linear basis method, the equation is derived as follows:

$$\frac{\partial^2\mu(x,t)}{\partial t^2} = C_w^2 \frac{\partial^2\mu(x,t)}{\partial x^2} \tag{6}$$

$$\iff \frac{\mu_n^{k+1} - 2\mu_n^k + \mu_n^{k-1}}{\Delta t^2} = C_w^2 \frac{\partial^2\mu_n^{k+1}}{\partial x^2}$$

$$-\frac{\partial^2\mu_n^{k+1}}{\Delta x^2} + \frac{1}{C_w^2\Delta t^2}\mu_n^{k+1} = \frac{1}{C_w^2\Delta t^2}(2\mu_n^k - \mu_n^{k-1}) \tag{7}$$

The detail of algorithm to solve (7) can be found in [4].

C. Analytic solution

The Eq.(1) and boundary and initial value Eq.(2) can also be given in analytically. Using separation of variables method, the analytical solution can be given as follow:

$$U(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos \frac{(2n-1)\pi c}{2L} t + B_n \sin \frac{(2n-1)\pi c}{2L} \right) \sin \frac{(2n-1)\pi x}{2L}. \tag{8}$$

Reader are encouraged to read some references for this method such as [6], [10], [12].

IV. RESULTS AND DISCUSSION

A. Rayleigh Ritz Piecewise-Linear Basis method

As shown in Eq.(7), it already fulfilled the form requirements for Rayleigh Ritz general equation, where the boundary conditions $y(0) = y(1) = 0$ are satisfied. However in this research, the boundary conditions are not zero,

$$y(0) = \alpha, \quad y(1) = \beta, \tag{9}$$

then the variable should be changed into:

$$z(x) = y(x) - \beta x - (1-x)\alpha. \tag{10}$$

Substitute (10) into (7), hence a new equation and boundary condition is acquired:

$$-\frac{d}{dx}\left(p(x)\frac{dz}{dx}\right) + q(x)z = f(x) - q(x)\beta(x), \quad 0 \le x \le 1 \tag{11}$$

$$z(0) = 0 \ , \ z(1) = 0$$

where:
- $p(x) = 1;$
- $q(x) = \frac{1}{C_w^2 \Delta t^2};$
- $f(x) = \frac{1}{C_w^2 \Delta t^2}(2\mu_n^k - \mu_n^{k-1});$
- $\beta(x) = \big(\frac{(4Vy)}{(L(2n-1)\pi c)}\big)\big(\frac{(2L)}{((2n-1)\pi)}\big)\Delta x.$

Moreover, the new boundary condition becomes:

$$0 \le x \le 1, y(0) = 0, y(1) = \big(\frac{(4Vy)}{(L(2n-1)\pi c)}\big)\big(\frac{(2L)}{((2n-1)\pi)}\big)\Delta x. \tag{12}$$

B. Comparison of Numerical Simulation using Rayleigh Ritz Method and Analytic solution

In order to see the robustness of Rayleigh Ritz Method, then the comparison of solution with analytical solution will be given. In this case the simulation will be given as the following conditions:
- Domain spatial: $0 \le \Delta x \le 1$.
- Time simulation (stability requirements): $0 \le \Delta t \le 10^{-6}$ (Δt depends on Δx).
- Steel as a material with modulus young $E = 20 \times 10^{10} N/m^2$.
- Gravitation: $G = 0.01$.
- Density: $\rho = 7850$.
- Total length: L= 705.
- Velocity v= $20 m/s$.
- Propagation of velocity $C_w = \frac{144 \times E \times G}{\rho}$.

Then using Fourier coefficient and (8) as shown in [12], the analytical solution of this problem is given as,

$$U(x,t) = \sum_{n=1}^{\infty}\left(B_n Sin\frac{(2n-1)\pi c}{2L}t\right)Sin\left(\frac{(2n-1)\pi x}{2L}\right)\left(\frac{4Vy}{L(2n-1)\pi c}\right)\left(\frac{(2L)}{(2n-1)\pi}\right). \tag{13}$$

The results of using numerical approach and analytical solution are given in Fig.4. The simulations at Fig.4 has axis notation(x), representing segment. The axis (y) representing displacement and axis (z) representing time. Those results describe the displacement on each time with the rage of segment between $0 \le L \le 1$ with $\Delta x = 0.2$. As it shown in Fig.4, the numerical Rayleigh Ritz results are in a good agreement with the analytical solution.

Moreover, several tests using different Δx value are elaborated in this research. The results can be seen as in the following L1 error table (shown in TABLE I). The detail of L1 norm error can be found in [13]. As shown in TABLE I, the optimal result is obtained using Δx = 0.02. With Δx smaller than Δx = 0.02 allows you to get smaller errors, but that computation will get longer because the discrete number increases.

To close this section, the comparision of numerical solution and analytical solution at segment [50,51] can be found in Fig. 5. It is also shown that numerical solution can approach the analytical solution well for sucker rod pump problem.

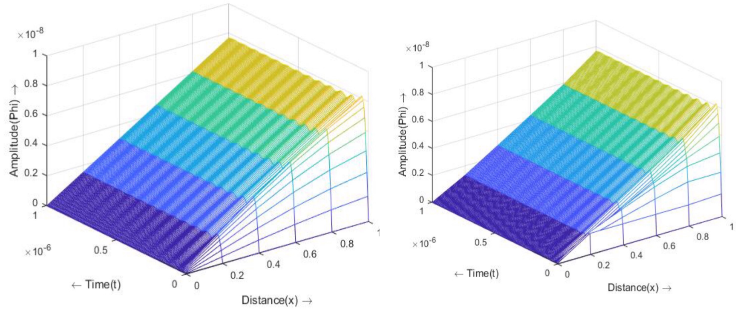

Fig. 4. Comparison Between Analytics (left) and Rayleigh Ritz (right)

TABLE I
Table error of simulation using several Δx.

M	Δx	L1 Error	Segment(x)	$length(t)$
1	0.2	2.00911 x 10^{-16}	6	184
1	0.05	3.30195 x 10^{-17}	21	734
1	0.02	1.45114 x 10^{-17}	51	1835

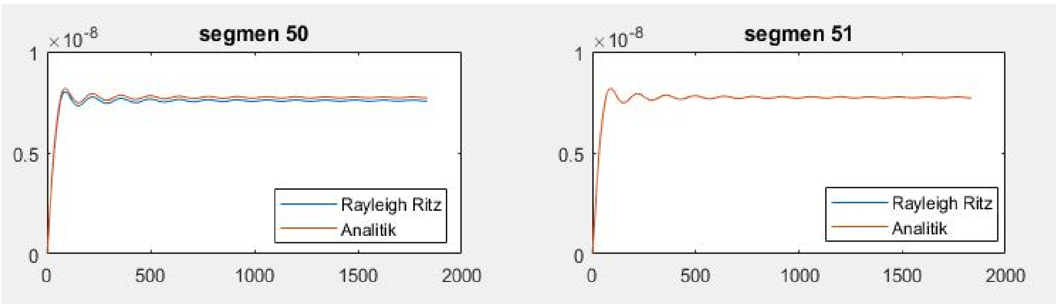

Fig. 5. Comparison Between Analytics and Rayleigh ritz at segments [50,51]

V. Conclusion

In this research the simulation of sucker rod pump problem using numerical approach Rayleigh Ritz approximation is presented. Based on the analysis of results, it can be concluded that Rayleigh Ritz Piecewise Linear can approach the solution well. The accuracy of the solution can be seen by comparing the numerical and analytical solution of the problem. The analytical results are conducted by separation of variables method. In this simulation, the L1 error rate is obtained 2.00911×10^{-17} at $\Delta x = 0.2$, 3.30195×10^{-17} at $\Delta x = 0.05$ and 1.45114×10^{-16} at $\Delta x = 0.02$.

References

[1] Annisa Aditsania, Silvy Dewi Rahmawati, Pudjo Sukarno, and Edy Soewono. Modeling and simulation performance of sucker rod beam pump. In *AIP Conference Proceedings*, volume 1677, page 080008. AIP Publishing, 2015.
[2] Robin Beckwith et al. Pumping oil: 155 years of artificial lift. *Journal of Petroleum Technology*, 66(10):101–107, 2014.
[3] RB Bhat. Natural frequencies of rectangular plates using characteristic orthogonal polynomials in rayleigh-ritz method. *Journal of Sound and Vibration*, 102(4):493–499, 1985.
[4] Richard L Burden and J Douglas Faires. Numerical analysis. *Cengage Learning*, 9, 2010.
[5] danel. Sucker Rod Pump (SRP). http://welloperation.blogspot.com/2015/01/sucker-rod-pump-srplengkap.html. Online; accessed 19 September 2018.
[6] Harumi Hattori. *Partial differential equations: methods, applications and theories*. World Scientific Publishing Company, 2013.
[7] Norman J Hyne. *Nontechnical guide to petroleum geology, exploration, drilling, and production*. PennWell Books, 2012.

[8] Zhongxiao Jia and GW Stewart. An analysis of the rayleigh–ritz method for approximating eigenspaces. *Mathematics of Computation*, 70(234):637–647, 2001.

[9] Xinfu Liu, Yaoguang Qi, Yanxiang Li, and Chunhua Liu. An approach to the design calculation of sucker rod pumping systems in coalbed methane wells. *Chinese Journal of Mechanical Engineering*, 24(2011), 2011.

[10] Marcus Pivato. *Linear partial differential equations and Fourier theory*. Cambridge University Press, 2010.

[11] Gordon D Smith. *Numerical solution of partial differential equations: finite difference methods*. Oxford university press, 1985.

[12] Walter A Strauss. *Partial differential equations: An introduction*. Wiley, 2007.

[13] F Muhammad Zain, M Garda Khadafi, and P H Gunawan. Analisis konvergensi metode beda hingga dalam menghampiri persamaan difusi. *E-Jurnal Matematika*, 7(1):1–4.