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Rayleigh Ritz Cubic Spline method for Displacement Simulation Sucker Rod

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Abstract

Artificial lift is one of the artificial mechanisms for lifting petroleum. This mechanism is used when the oil cannot flow due to a decrease in pressure on the well hole. Sucker beam rod pump is one type of artificial lift. The research aims at analyzing the design of the pumping system based on stretching or displacement from the shaft rod. Dick rod is one component of the system of shaft rod pump which is located in mining wells. These rods function as a place for the dependence of oil. Displacement or stretch is modeled as a wave equation. Numerical calculations are performed to determine the sucker rod and displacement solutions. The solution of the same displacement at the next stage can be used as additional information for sucker rod beam pump operator to determine the condition of the rod in the well. In this research the Rayleigh Ritz method is used to determine the numeric solution of the equation, the function of B-Spline Base is used in the RayleighRitz method because the B-Spline Base function is more flexible in approaching solutions than linear functions. The basic function of B-Spline Basics is the function of Cubic Spline Interpolation. The solution obtained from this numerical calculation is a matrix that shows the extent of the sucker rod segment with time. The results obtained have L-1 errors 1.24×10^{-14} and L-2 error 1.08×10^{11}.

Keywords: sucker rod, pump system, finite difference method, different element method, rayleigh ritz, petroleum.

Abstrak

Artificial lift adalah salah satu mekanisme pengangkatan buatan minyak bumi. Mekanisme ini digunakan bila minyak sudah tidak dapat mengalir akibat menurunnya tekanan pada lubang sumur. *Sucker beam rod pump* adalah salah satu jenis pengangkatan buatan. Penelitian bertujuan menganalisis desain sistem pemompaan berdasarkan renggangan atau *displacement* dari *sucker rod*. *Sucker rod* adalah salah satu komponen dari sistem *sucker rod beam pump* yang terletak dalam sumur penambangan. Batang ini berfungsi sebagai tempat bergantungnya muatan minyak. *Displacement* atau renggangan dimodelkan sebagai persamaan gelombang. Perhitungan numerik dilakukan untuk menentukan solusi persamaan *displacement sucker rod*. Solusi dari persamaan *displacement* ini pada tahap berikutnya dapat digunakan sebagai informasi tambahan bagi operator *sucker rod beam pump* untuk mengetahui kondisi dari *sucker rod* di sumur. Pada penelitian ini metode Rayleigh Ritz digunakan untuk menentukan solusi numeerik persamaan tersebut, fungsi *B-Spline Basis* dipakai dalam metode Rayleigh Ritz karena fungsi *B-Spline Basis* lebih fleksible dalam mendekati solusi dibandingkan fungsi linear. Fungsi dasar dari *B-Spline Basis* merupakan fungsi *Cubic Spline Interpolation*. Solusi yang didapat dari perhitungan numerik ini adalah matriks yang menunjukan perenggangan terhadap segmen sucker rod terhadap waktu. Hasil yang didapat memiliki galat L-1 1.24×10^{-14} dan galat L-2 1.08×10^{11}.

Kata Kunci: sucker rod, sistem pompa, metode beda hingga, metode beda elemen, rayleigh ritz, minyak bumi.

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I. Introduction

OIL drilling causes the pressure in the oil well hole relatively high, thus oil can flow by itself at the beginning [1]. But over time, the well's pressure decreases so that the oil can no longer flow to the surface by itself [1]. So the oil miners are looking for ways to mine the oil that is still in the well, that is by artificial lifting. There are various artificial lifting mechanisms, among them are gas lift, electric submersible pump, hydraulic pump, sucker rod pump, etc [7].

In this research, sucker rod beam pump is chosen to be modeled because it has been used for more than 150 years, because it has the advantages of simple structure, durable, easy maintenance and so on [2]. Sucker rod beam pump has a part called sucker rod as a place to hang a tube that will be filled with oil or commonly called a plunger. Sucker rod is made of metal material. It is relatively small. Metal material also has elasticity. Elasticity can make the sucker rod stretch and shrink if given a load and exposed to hot or cold temperatures.

In previous research the strain of the sucker rod has been examined. The research shows a stretch of the sucker rod following the wave equation which has initial conditions and boundaries [9], the research used finite difference method. Rayleigh Ritz Cubic Spline is one type of Finite Element method that will be used in this research. Rayleigh Ritz is chosen because no one has used this method for sucker rod problems.

II. Literature Review

A. Mathematical model of sucker rod problem

In research [9], displacement of sucker rod when operating is modeled following the wave equation with initial conditions and boundaries as follows :

$$\frac{\partial^2 \mu(x,t)}{\partial t^2} = C_w^2 \frac{\partial^2 \mu(x,t)}{\partial x^2}, \tag{1}$$

with initial and boundary conditions:

$$\mu(x,0) = 0, \frac{\partial \mu(x,0)}{\partial t} = \frac{Vy}{L}, \mu(0,t) = 0, \frac{\partial \mu(L,t)}{\partial x} = 0, \tag{2}$$

where $(\mu)(x,t)$ is displacement of sucker rod, (C_w) is the propagation velocity, (V) is the relative velocity of the lower end of rod string, (L) is the total length of string, and y is the number of sucker rod segments.

Research about sucker rod starting in 1969 by Roy M. Knapp [8] about dynamic investigation of sucker rod pumping, the goal of the research is to show the feasibility of using the mathematical finite model to simulate and analyze pump systems. Research that focuses on developing models and applications to optimize design sucker rod for mining carried out by S. Miska [10]. The performance of sucker rod string is one of the most important factors in the sucker rod beam pump system. S.G. Gibbs [6] build models sucker rod system for computer applications. The model developed by S.G. Gibbs [10] uses a one-dimensional damped wave equation to predict performance from sucker rod string. The system model for computer applications was obtained by S.G. Gibbs [6] used Finite Difference method.

Research about the flexural vibration of the rectangular plate system by J. Yuan [12] uses spring artificial in the Rayleigh Ritz method. The research examined free vibration on iron plates. Variations of the Ritz Rayleigh method have been studied for double well quantum anharmonic oscillators [3]. R.F. Bishop [3] focused his research on variational calculations of single and multistate conditions based on orthogonal (uncorrelated) or non-coogonal (correlated) bases on double well quantum anharmonic oscillators. The Rayleigh Ritz variant calculation for the surface of the original metal will be compared with the experimental value of polycrystalline metal by V. Sahni [11].

This research uses the same storyline as the research conducted by LIU Xinfu [9], which is solving the equation (1) with the Rayleigh Ritz Cubic Spline method. This research aims at implementing the Rayleigh Ritz Cubic Spline method to simulate sucker rod movement, and simulates the results of the

Rayleigh Ritz Cubic Spline method. The results of the Rayleigh Ritz method will be compared with analytic results, and calculated the error.

B. Sucker Rod Beam Pump

Equipment on the surface are primer mover converts electrical energy into rotational motion crank. The rotation motion crank causes the pitman to spin, The up and down movement from walking beam forwarded to horse head becomes upstroke and downstroke sucker rod string. [5]. The illustration can be seen in the Fig. 1.

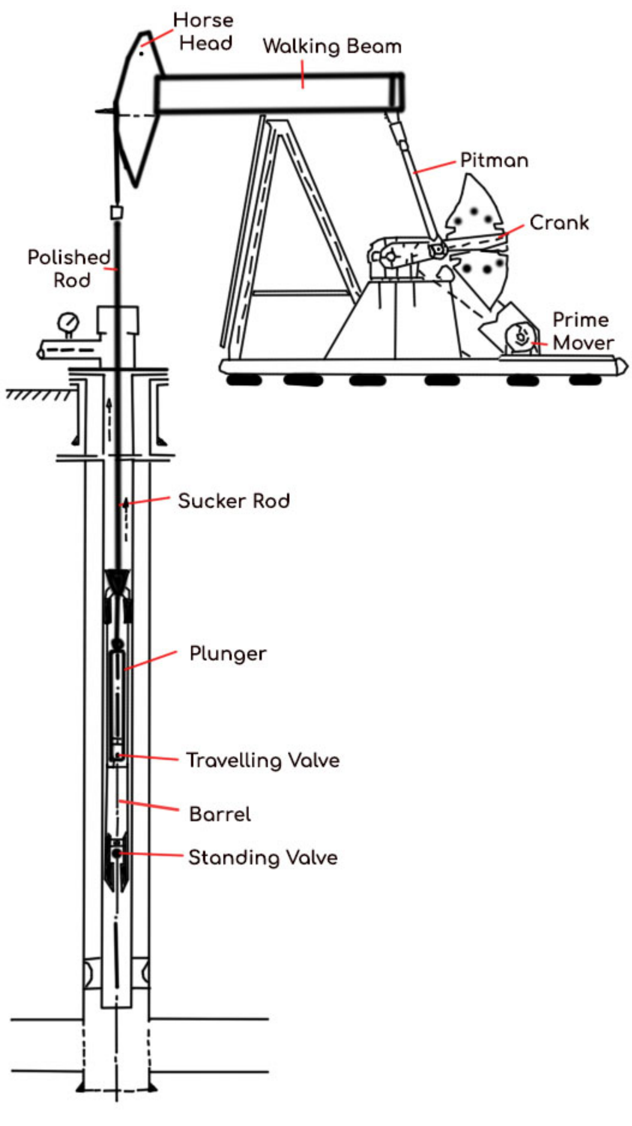

Fig. 1. The illustration of Sucker Rod Beam Pump

In the downhole, there are 4 main components: Plunger is a long piston moves up and down and also functiones to transporting the oil (fluid) from the bottom of the well to the tubing column up to the surface. Plunger is moving up and down in the cylindrical tube called working barrel. Traveling valve, which is a ball-shaped valve, that opens and closes in the plunger and standing valve which is a ball-shaped valve, that opens and closes in the downhole. The traveling valve closer to the standing valve causes pressure to increase between the standing valve and traveling valve, so that the traveling valve opens and the standing valve closes, this allows the oil to enter the plunger. When upstore the pressure between standing valve and travelling valve decreases and the pressure between traveling valve and the ground surface increases so standing valve opens allowed oil to fill the downhole and traveling valve closes so that the oil already in the sucker rod string does not spill back into the downhole. The illustration of traveling valve and standing valve can be seen in the Fig. 2.

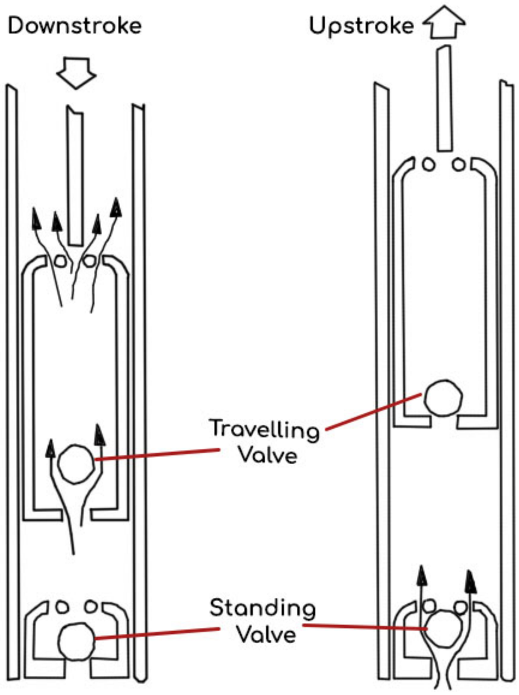

Fig. 2. Upstroke and downstroke illustration.

C. Rayleigh Ritz Method

The Rayleigh Ritz method is a method for solving ordinary differential equations with the equation model as follow [4] :

$$-\frac{\mathrm{d}}{\mathrm{dx}}\left(p(x)\frac{\mathrm{dy(x)}}{\mathrm{dx}}\right)+q(x)y(x)=f(x),\ \text{for } 0\leq x\leq 1, \tag{3}$$

and the boundary conditions are given as,

$$y(0)=y(1)=0. \tag{4}$$

The differential equation describes the deflection $y(x)$ represented by $q(x)$. The deflection caused due $p(x)$ and $f(x)$ [4]. The solution $y(x)$ is approximated by numeric by $\phi(x)$. The $\phi(x)$ function can be linear, parabolic, b-spline, etc. In this research the Rayleigh Ritz B-Spline Basis method is used to determine the equation (3) solution. The difference between the Rayleigh Ritz B-Spline Basis method and the other lies in the function $\phi(x)$. B-Spline Base can differentiate (3) on $C_0^2[0,1]$. A new basis functions is created for B-Spline Basis. These basis functions are similar to the cubic spline interpolation, the function is $S(x)$ with $S\in C_0^2[-\infty,\infty]$. The detail of $S(x)$ and $\phi(x)$ functions can be found in [4]. Reader is encouraged to read [4] for more detail about this method.

III. RESEARCH METHOD

A. Research Flowchart

This research starts from collecting data obtained from researches [9] and related books. After getting the data, the equation (1) will be discretized to create a discrete model of the Rayleigh Ritz method. After the discretization model, the program is made to perform Ritz Rayleigh calculations and Analytical calculations. The results of the calculation of the Rayleigh Ritz method will be compared with the results of Analysis. The research flowchart is explained in Fig. 3 :

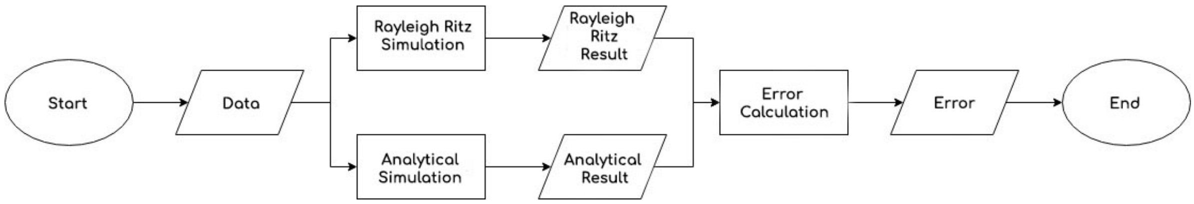

Fig. 3. Research flowchart

B. Discretization of Wave Equations

The equation (1) must be formed into (3) so that it can be solved using the Rayleigh Ritz method. The discretization of (1) starts from semi-discretization for time only, thus,

$$-\frac{d^2\mu_n^{k+1}}{dx^2}+\frac{1}{C_w^2\Delta t^2}\mu_n^{k+1}=\frac{1}{C_w^2\Delta t^2}(2\mu_n^k-\mu_n^{k-1}) \tag{5}$$

where,

$$p(x)=1\ ,\ q(x)=\frac{1}{C_w^2\Delta t^2}\mu_n^{k+1},\ \text{dan}\ f(x)=\frac{1}{C_w^2\Delta t^2}(2\mu_n^k-\mu_n^{k-1}). \tag{6}$$

The equation (5) and (6) is the semi-discrete form of (1) that has similar form as (3). In the modification of equation (1), Rayleigh Ritz is used only for solving spatial while for time it is solved with Finite Difference. Next the cubic spline Rayleigh-Ritz algorithm by [4] in page 707 can be implemented.

IV. RESULTS AND DISCUSSION

A. Rayleigh Ritz Solution

Simulation displacement sucker rod is obtained by solving (1) boundary conditions and initial conditions (2). However, Rayleigh Ritz has boundary conditions $y(0)=y(1)=0$. Then in this research, the boundary condition must be converted as follows, if the boundary condition satisfying,

$$y(0)=\alpha\ ,\ y(1)=\beta, \tag{7}$$

Then, variable y should be changed into:

$$y=z+\beta x+(1-x)\alpha \tag{8}$$

Substituting (8) into (1) hence a new equation and boundary condition is acquired :

$$-\frac{d}{d(x)}\left(p(x)\frac{dz}{dx}\right)+q(x)z=f(x)-q(x)\beta(x),\ \text{for}\ 0\le x\le 1 \tag{9}$$

$$z(0)=0\ ,\ z(1)=0$$

Substituting $p(x)$, $q(x)$, and $f(x)$ from (6) into (9) with $z=U^{k+1}$ can be obtained :

$$p(x) = 1 \ , \ q(x) = \frac{1}{C_w^2 \Delta t^2} \text{ dan } f(x) = \frac{2u_{n+1}^k - u_{n+1}^{k-1}}{C_w^2 \Delta t^2} - \frac{1}{C_w^2 \Delta t^2} \times ... \tag{10}$$

$$\frac{4Vy}{C_w L^2}\left(1 - Cos(\frac{\pi}{2L})\right) Sin\left(\frac{\pi c_w t}{2L}\right) Sin\left(\frac{\pi}{2L}\right)$$

and the boundary condition:

$$y(0) = 0 \text{ dan } y(1) = \frac{4Vy}{C_w L^2}\left(1 - Cos(\frac{\pi}{2L})\right) Sin\left(\frac{\pi c_w t}{2L}\right) Sin\left(\frac{\pi}{2L}\right)(\omega t) \tag{11}$$

B. Analytical Solution

The equation (1) with initial conditions and boundaries conditions (11) can be solved analyticaly, using separation variables, where it is assumed that $U(x,t) = \psi(x)h(t)$ is a solution to the wave equation (1), then two ordinary differential equations can be obtained as,

$$\psi(x)\frac{\partial^2 h}{\partial t^2} = C_w^2 h(t)\frac{\partial^2 \psi}{\partial x^2}, \tag{12}$$

$$\frac{1}{C_w^2}\frac{1}{h}\frac{\partial^2 h}{\partial t^2} = \frac{1}{\psi}\frac{\partial^2 \psi}{\partial x^2}, \tag{13}$$

$$\frac{\partial^2 h}{\partial t^2} = -\lambda C_w^2 h \text{ and } \frac{\partial^2 \psi}{\partial x^2} = -\lambda \psi \tag{14}$$

With the coefficient Fourier technique, the final analytical solution can be given as follows,

$$U(x,t) = \left(\frac{4Vy}{(2n-1)\pi c L}\right)\left(\frac{2L}{(2n-1)\pi}\right) Sin\frac{(2n-1)\pi x}{2L}. \tag{15}$$

C. Rayleigh Ritz Simulation

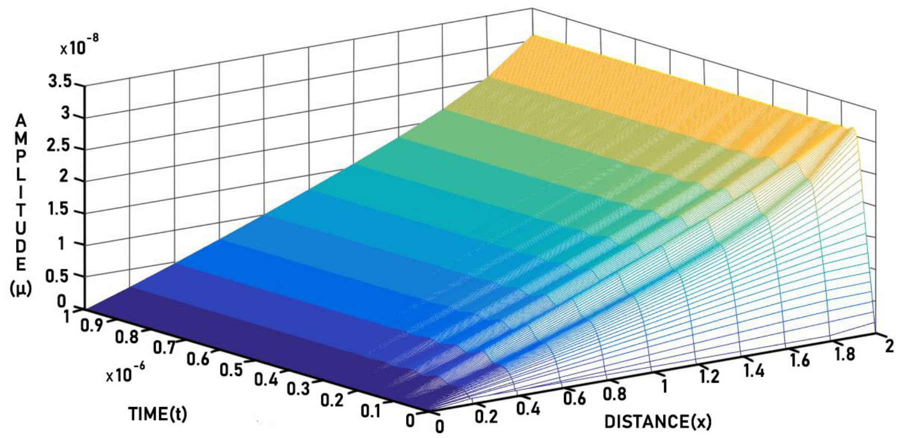

Fig. 4. Rayleigh Ritz on $0 \leq x \leq 2$ with $\Delta x = 0.2$

Fig. 4 is the result of Rayleigh Ritz method in the simulation of sucker rod displacement problem. The graph shows the displacement at time in $0 \leq t \leq 10^{-6}$ with $\Delta t = \Delta x/m \times C_w = 2.7257 \times 10^{-9}$ on $0 \leq x \leq 2$ with $\Delta x = 0.1$.

Sucker rod in this simulation uses steel as the material with modulus young $E = 20 \times 10^{10}\ N/m^2$, density $\rho = 7850\ kg/m$ and Relative velocity $V = 20\ m/s$. From Fig. 4, it can be seen that over time the graph will be constant at a certain value. This is because in this study only observed in $0 \leq x \leq 2$.

Meanwhile the overall length of the domain $L = 705$ meters, so that $0 \leq x \leq 2$ is the beginning of the sucker rod which is bound to polished rod.

The vibration produced at the bound state is not large compared to the vibration at the unbound state. There are 4 phases of oil lifting: the deformation period of the rod string and tubing (when will upstroke). The upstroke period, the deformation period of the rod string and tubing(when will downstroke), the downstroke period. the vibrations caused by the load during movement of the upstroke and downstroke are more stable so that the resulting displacement is small.

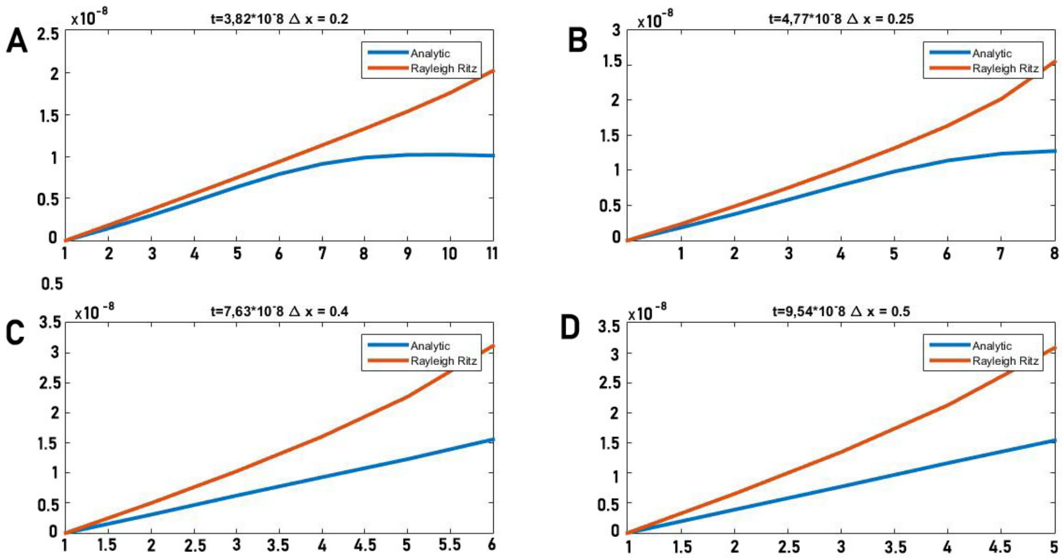

Fig. 5. comparison between Rayleigh-Ritz and Analytic with $0 \leq x \leq 2$ and $\Delta x = 0.2(A); 0.25(B); 0.4(C); 0.5(D)$ on the fourteenth iteration.

Fig. 5 is the result of analytic solutions compared with Rayleigh Ritz on all segments at time (t = $14\Delta t$) with different Δx. Figure 5 shows that numerical solutions are able to approach analytical solutions for all Δx. Based on the chart A and B in fig. 5, analytical solutions are decreasing in the last segment while numerical solutions are still moving up. On graphs C and D, both analytical and numerical solutions are increasing, but the addition value of numerical solutions is greater than the analytic.

In this study, the domain length should be $0 \leq x \leq L$, but only $0 \leq x \leq 2$ are observed. The matrix will get bigger for each length increment, thus the selection of Length and Δx is limited due to the finite resources for calculations.

TABLE I
THE TOTAL ERROR FOR SOME SIMULATIONS

Δx	Δt	L-1 Error	L-2 Error
0.2	2.73E-09	1.2406E-14	1.08E-11
0.25	3.41E-09	1.3111E-14	1.13E-11
0.4	5,45E-09	1.5049E-14	1.26E-11
0.5	6.81E-09	1.6036E-14	1.33E-11

Here several tests using different Δx are elaborated. The error results for each different Δx can be seen in TABLE I. As shown in TABLE I, decreasing Δx then the error will also decreasing. So that for more optimal results is at $\Delta x = 0.2$. Smaller error is obtained if Δx smaller than $\Delta x = 0.2$, when Δx get smaller the computation will get longer.

V. Conclusion

In this study, a simulation of the sucker rod displacement is approached with the Ritz and Analytical Rayleigh method. The results obtained by the Rayleigh Ritz method successfully approaches analytic solutions on domain $0 \leq x \leq 2$ at each Δx. The L-1 error obtained in this simulation is 1.24×10^{14} for Δx = 0.2, 1.3×10^{14} for Δx = 0.25, 1.5×10^{14} for Δx = 0.4, 1.6×10^{14} for Δx = 0.5. The L-1 error obtained in this simulation is 1.08×10^{11} for Δx = 0.2, 1.13×10^{11} for Δx = 0.25, 1.26×10^{11} for Δx = 0.4, 1.33×10^{11} for Δx = 0.5. The optimal results are obtained at Δx = 0.2 with an L-1 error of 1.24×10^{14} and L-2 error 1.08×10^{11}. Smaller Δx will produce smaller errors, but the computation will be longer.

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