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Stock Portfolio Optimization on JII Index using Multi-Objective Mean-Absolute Deviation-Entropy

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Abstract

Stock portfolio optimization is allocating stock assets from investors to manage return and risk. Investors need a high-return portfolio with a given level of risk, and portfolio optimization can help to find the feasible one. The data used for this problem are stocks listed on the Jakarta Islamic Index (JII). The portfolio optimization methods are applied Multi-Objective Mean-Absolute Deviation (MAD) and Entropy. MAD is used because it can solve the portfolio optimization problem for the nonnormal distribution of data. Meanwhile, entropy is used because it can better diversify the weight of stocks in the MAD portfolio. The Equal Weight portfolio represents a portfolio with the maximum entropy. The portfolio divides into four scenarios (3, 5, 7, and 26 stocks). Experiment results in this study show that MAD-Entropy and Equal Weight portfolio outperform the MAD portfolio in Sharpe Ratio and Performance Ratio. MAD only excels in one period, influenced by a stock that has a fantastic return in a certain period.

Keywords: portfolio, mean-absolute deviation, entropy, Sharpe ratio, performance ratio

I. INTRODUCTION

A portfolio combines investors' financial assets, such as stocks, bonds, and mutual funds, to achieve their financial goals. A stock portfolio is a collection of several company stocks owned by an investor. Investors need a portfolio with high returns at a given level of risk. Portfolio optimization can help find it by optimizing the weight (proportion) of stocks in the portfolio.

Stock portfolio optimization allocates stock assets from investors to manage returns and risks. Managing the portfolio problem leads to the optimization model, and various approaches have been proposed, as discussed by [1], [2].The basic theory of portfolio optimization was first introduced by Markowitz using the Mean-Variance (MV) model that selects a portfolio by minimizing risk at the level of return targeted by investors [3]. The variance of portfolio returns represents the risk of the MV portfolio.

MV model assumes that the return on assets has a normal distribution. However, this assumption is not always fulfilled. Handling this issue, Konno and Yamazaki [4] introduced the Mean-Absolute Deviation (MAD) model by replacing variance with absolute deviation as a risk measurement for the portfolio. Erdas [5] has also used MAD for stock indices in Turkey. A limit on trading volume is added to the model to anticipate the systematic risk. The principle of the MAD portfolio is the same as the MV portfolio, maximize return at a given level of risk or minimize risk at a level of desired return. MAD is used because it can solve the portfolio optimization

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problem for the nonnormal distribution of data. Unlike MV, which deals with quadratic programming, the computation in the MAD deals with linear programming. MAD offers less computational difficulty than the MV, especially for a portfolio with many stocks. However, the MAD portfolio frequently faces a diversification problem where specific stocks dominate the portfolio's weight. Some researchers proposed entropy to handle the diversification problem.

Li and Zhang [6] examined the mean-variance-entropy by looking at the effect of diversification on portfolio optimization. Lu et al. [7] investigated portfolio optimization using the entropy principle. Lam et al. [3] proposed including entropy in the MAD portfolio to keep portfolio diversity since the greater the entropy value of the portfolio, the greater the diversification of the portfolio. Using stocks in the Dow Jones Industrial Average, Lam et al. show that the proposed MAD-Entropy model outperforms the MAD and Equal Weight portfolio.

This study aims to implement the MAD-Entropy portfolio to stocks on the Indonesian stock exchange. The selected stocks in the portfolio are stocks in the Jakarta Islamic Index (JII). How is the performance of the MAD-Entropy portfolio compared to the model that only uses MAD and Equal Weight on the JII stock index? How does the number of shares in the portfolio affect? How do these portfolios perform when the market is normal, sideway, and experiencing turbulence due to the COVID-19 pandemic?

I. LITERATURE REVIEW

Various research approaches have been carried out for portfolio optimization. One is Geoffrey Kasenbacher et al. [8] with the title Mean-Variance vs. Mean-Absolute Deviation: A performance Comparison of Portfolio Optimization Models. This study discusses the performance of two portfolio optimization methods, MV and MAD. The data used are stocks listed on the American stock index, the S\&P 500. This study aims to determine whether the MAD model can beat the MV model in terms of the resulting returns. Using the Sharpe Ratio, the MAD model beat the MV model for 5 out of 6 cases. The MAD model gets a higher return than the MV model with the same level of risk.

Mehmet Aksarayli and Osman Pala [9] conducted another research related to portfolio optimization with the title A Polynomial Goal Programming Model for Portfolio Optimization based on Entropy and Higher Moments. This study discusses how to optimize portfolios by involving high moments and entropy. Two entropy measures: Shannon entropy and Gini-Simpson entropy, are applied in this study. The data used are shares listed on the Turkish stock exchange. This study compares the previous model with a model that can handle data with the nonnormal distribution. The study showed that the Gini-Simpson entropy performs better than the existing model. However, the Shannon entropy provides a more diversified portfolio than the Gini-Simpson entropy.

Weng Siew Lam et al. [3] investigated portfolio optimization by combining MAD and entropy. This study aimed to find a portfolio by minimizing risks as in the MAD method and maximizing entropy. The model representation of this approach is a Multi-Objective Optimization. Using stocks data listed on the New York Stock Exchange (NYSE), the combination of MAD and entropy model produces better performance and a higher average return than the model that uses only MAD.

II. RESEARCH METHOD

In conducting this study, it is necessary to collect stock price data on the Indonesia Stock Exchange from January 1, 2010, to December 31, 2021 (624 weeks) from the yahoo website. The details of the stocks used are explained in section IV. The collected data is subsequently cleaned and prepared for processing, including the handling of missing data, in a procedure known as pre-processing. The process continues with the return calculation, the MAD value, and entropy calculation. The flow chart of this research process can be seen in Fig. 1.

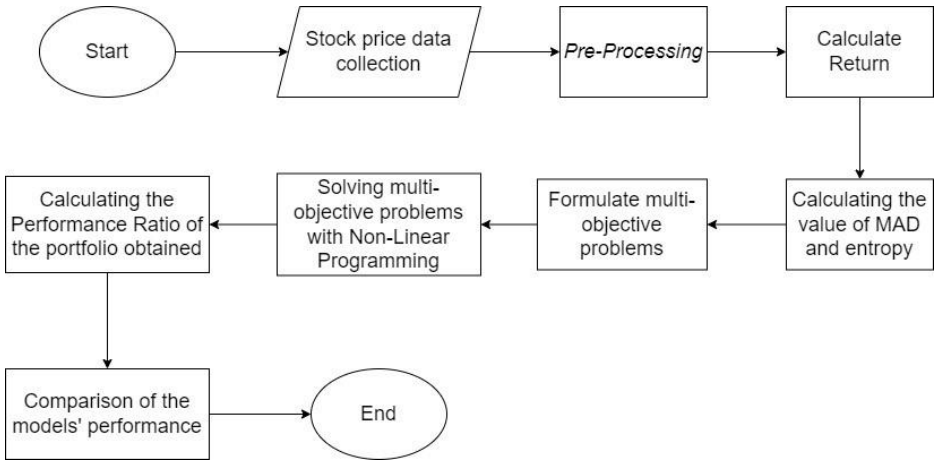

Fig. 1 Process Chart of Stock Portfolio Optimization on JII Index

Non-linear programming is then used to find the portfolio weight. The ratio value of the portfolio is also computed from the previous phase to determine which stocks have the best performance. The best model to utilize may be seen from each performance ratio achieved.

A. Stocks Return

Stock return calculation is using (1)

$$R = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

where R is the return, P_t is the stock price at time t, and P_{t-1} is the stock price at time $t-1$.

B. Mean-Absolute Deviation model

The absolute deviation risk function has been introduced by Konno and Yamazaki [4] to replace the risk calculation using the standard deviation of returns in the Markowitz model

$$w(\boldsymbol{x}) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right] \tag{2}$$

The MAD portfolio is a portfolio with a weight that gives the maximum value to the MAD function (2). During period a portfolio using the MAD model is the solution to the optimization problem that can be solved by using linear programming [3]

$$\min_{\boldsymbol{x}} \frac{\sum_{t=1}^{T} y_t}{T}, \tag{3}$$

subject to

$$y_t + \sum_{j=1}^{n} a_{jt} x_t \geq 0, t = 1, \ldots, T, \tag{4}$$

$$y_t - \sum_{j=1}^{n} a_{jt} x_t \geq 0, t = 1, \dots, T, \quad (5)$$

$$\sum_{j=1}^{n} x_j = 1, \quad (6)$$

$$x_j \geq 0, j = 1, \dots, n, \quad (7)$$

$$\sum_{j=1}^{n} E[R_j] x_j \geq \rho. \quad (8)$$

Variable R_j and x_j are return and portfolio weight of stock j, ρ is target of return portfolio desired by the investor. The value of r_{jt} is the realization of R_j, r_j is the average return of stock j and

$$a_{jt} = r_{jt} - r_j. \quad (9)$$

C. Mean-Absolute Deviation and Entropy model

$H(x)$, as the uncertainty value, was first introduced by Shannon [10] in previous research. The main advantage of using entropy is that it creates a well-diversified portfolio. As shown in (10), the entropy is a function of portfolio weights $x_1, \dots, x_n$, with a maximum value of $\ln n$. The maximum value can be reach when the value of $x_j = \frac{1}{n}, j = 1, 2, \dots, n$. The portfolio with such a weight is known as the Equal Weight portfolio.

$$H(\boldsymbol{x}) = -\sum_{j=1}^{n} x_j ln\, x_j \quad (10)$$

The entropy function also overcomes the problem of extreme weighting on one stock in the portfolio, which may lead to a diversification problem. A higher entropy value of a portfolio means higher portfolio diversification. Diversification can reduce the risk to the portfolio.

Portfolio optimization using the MAD-Entropy approach is an optimization problem with three objectives: maximizing the average return, minimizing the absolute deviation, and maximizing the entropy. Let R^* and W^* be the maximum average return and minimum absolute deviation for every objective, respectively. Rewriting the optimization problem into (11)-(16) [3], [9], [11], the MAD-Entropy portfolio weight is calculated by using nonlinear programming approach.

$$\min_{\boldsymbol{x}} Z = \left|\frac{d_1}{R^*}\right| + \left|\frac{d_2}{W^*}\right| + \left|\frac{d_3}{\ln n}\right|, \quad (11)$$

subject to

$$\sum_{j=1}^{n} x_j = 1, \quad (12)$$

$$x_j \geq 0, j = 1, \dots, n, \quad (13)$$

$$R(\boldsymbol{x}) + d_1 = R^*, \quad (14)$$

$$w(\boldsymbol{x}) - d_2 = W^*, \tag{15}$$

$$H(\boldsymbol{x}) + d_3 = \ln n\,. \tag{16}$$

Variable $d_1, d_2,$ and d_3 are deviation of portfolio mean return, absolute deviation and entropy from the optimal value.

D. Model Performance

The model's performance can be measured using the ratio of portfolio mean performance and the absolute deviation expressed by the equation (17). A higher performance ratio means a higher average return or lowers the absolute deviation. Similar to the performance ratio, the Sharpe ratio is also calculated to measure the model's performance: a ratio of the average return and standard deviation return.

$$Performance\ ratio = \frac{R(\boldsymbol{x})}{w(\boldsymbol{x})}. \tag{17}$$

III. RESULTS AND DISCUSSION

The system is run in python. The data to process contain 17,810 weekly stock returns. The market cap of each stock is shown in Table I. This study compares three methods: MAD, Equal Weight, and MAD-Entropy. Four scenarios are differentiated based on the number of stock portfolios: three stocks, five stocks, seven stocks, and 26 stocks. The stock selection is determined based on the market cap value from the highest to the lowest. The model's performance is measured using these scenarios for 2016-2017, 2018-2019, and 2020-2021. The result of standardized data can be seen in Table I.

Table I Stocks Data

(a) FIRST 13 STOCKS		(b) LAST 13 STOCKS	
Code	**Market Cap (Billions)**	**Code**	**Market Cap (Billions)**
TLKM	448,752	INTP	40,125
TPIA	207,087	SMGR	39,296
UNVR	130,092	PTBA	38,249
UNTR	96,237	PGAS	34,423
CPIN	93,059	EXCL	28,957
ADRO	89,561	PWON	23,695
ICBP	87,464	TKIM	22,415
BRPT	82,029	SCMA	20,543
KLBF	77,813	JPFA	18,880
INCO	63,593	AKRA	17,765
ANTM	60,077	MNCN	13,921
INDF	52,463	KAEF	9,747
INKP	44,178	WIKA	8,791

A. 3 stocks

Table II Statistics of 3 stocks

Method	Mean Return			Standard Deviation		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.00644	0.0005	**-0.0014**	**0.02184**	0.0232	**0.0396**
Equal Weight	0.00949	**0.0017**	-0.0018	0.02250	0.0236	0.0397
MAD-Entropy	**0.00951**	0.0011	-0.0019	0.02255	**0.0227**	0.0397

Table III Ratio of 3 stocks

Method	Sharpe Ratio			Performance Ratio		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.29482	0.0210	**-0.0351**	0.40838	0.0253	**-0.0519**
Equal Weight	**0.42197**	**0.0734**	-0.0441	**0.55934**	**0.0907**	-0.0639
MAD-Entropy	0.42145	0.0478	-0.0468	0.55798	0.0577	-0.0691

Table II and Table III show the comparison of stock portfolio optimization performance using the MAD, Equal Weight, and MAD-Entropy methods for 2016-2017, 2018-2019, and 2020-2021. MAD-Entropy is much superior to MAD in the 2016-2017 period and slightly superior to Equal Weight (EW) in terms of Mean Return. However, due to less return standard deviation, the MAD portfolio risk is better than EW and MAD-Entropy. For Performance Ratio, Equal Weight slightly outperforms MAD-Entropy. In the 2018-2019 period, Equal Weight far exceeds MAD-Entropy and MAD from Sharpe Ratio and Performance Ratio with 0.073 and 0.09. 2020-2021 is a turbulent period due to the Covid-19 pandemic in Indonesia. Almost all of the stocks on the Indonesia Stock Exchange dropped. During this period, the Mean Return of the MAD portfolio is better than EW and MAD-Entropy.

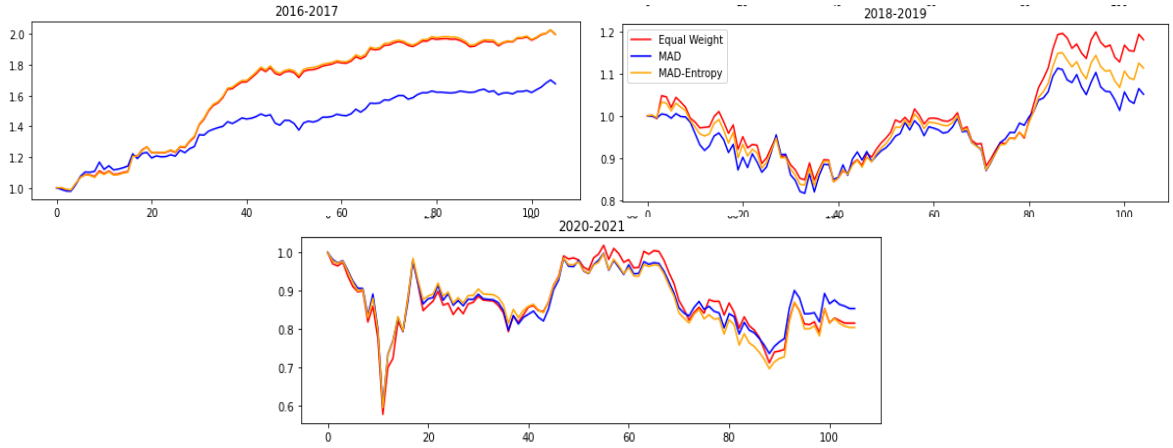

Fig. 2 Portfolio growth of 3 stocks

Fig. 2 shows that the growth of portfolio value from the Equal Weight and MAD-Entropy methods much outperforms MAD in the 2016-2017 period. During the 2018-2019 period, the portfolio growth of the EW exceeded both MAD-Entropy and MAD. Moreover, in the 2020-2021 period, the MAD produces a bit higher portfolio value.

B. 5 stocks

Table IV Statistics of 5 stocks

Method	Mean Return			Standard Deviation		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.00639	0.00002	-0.00097	**0.02018**	**0.02112**	0.03827
Equal Weight	0.00805	**0.00201**	-0.00047	0.02206	0.02395	**0.03815**
MAD-Entropy	**0.00832**	0.00155	**-0.00042**	0.02200	0.02474	0.03910

Table V Ratio of 5 stocks

Method	Sharpe Ratio			Performance Ratio		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.31653	0.0009	-0.0253	0.43978	0.0010	-0.0377
Equal Weight	0.36482	**0.0838**	-0.0124	0.49683	**0.1054**	-0.0188
MAD-Entropy	**0.37801**	0.0626	**-0.0107**	**0.50918**	0.0782	**-0.0163**

Table IV and Table V shows that MAD-Entropy has a higher mean return than the EW and MAD in the 2016-2017 period. However, in that period, the MAD offers less standard deviation, which means it offers a lower risk portfolio. Moreover, the performance ratio of MAD-Entropy is higher than the EW and MAD. In the 2018-2019 period, the EW gets the highest mean return value and performance ratio compared to MAD-Entropy and MAD. During the turbulence period in 2020-2021, the MAD-Entropy offers a bit higher mean return and performance ratio than the EW and MAD.

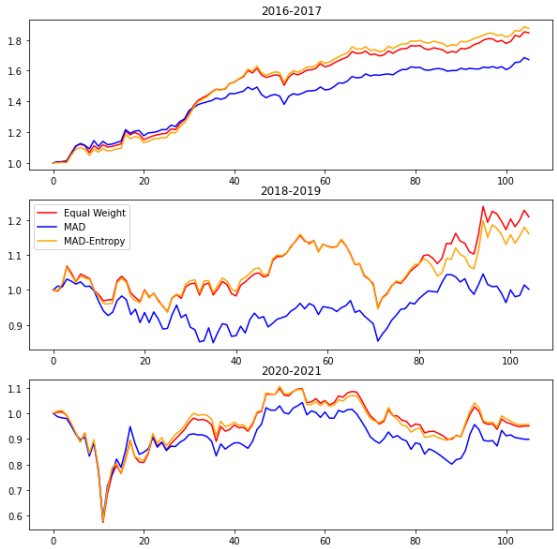

Fig. 3 Portfolio growth of 5 stocks

Fig. 3 shows the growth of a portfolio of 5 stocks across all periods. In 2016-2017, MAD-Entropy had the highest portfolio value, surpassing Equal Weight and MAD. In the 2018-2019 period, the portfolio growth of the Equal Weight method outperformed MAD-Entropy and MAD. However, the graph movement is almost the same. In the 2020-2021 period, the three methods suffer a negative return with MAD-Entropy and Equal Weight, which has a slight loss compared to MAD.

C. 7 stocks

Table VI Statistics of 7 stocks

Method	Mean Return			Standard Deviation		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.00565	0.0005	-0.0008	**0.01980**	**0.0190**	**0.0345**
Equal Weight	0.00828	**0.0018**	0.0004	0.02263	0.0242	0.0388
MAD-Entropy	**0.00909**	0.0014	**0.0005**	0.02472	0.0258	0.0396

Table VII Ratio of 7 stocks

Method	Sharpe Ratio			Performance Ratio		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.28514	0.0285	-0.0244	0.39817	0.0345	-0.0368
Equal Weight	0.36576	**0.0727**	0.0110	**0.49819**	**0.0921**	0.0177
MAD-Entropy	**0.36760**	0.0540	**0.0121**	0.49229	0.0686	**0.0197**

Table VI and Table VII show that MAD-Entropy has an outstanding return in the 2016-2017 period. However, the MAD's portfolio standard deviation is lower, which means it has less risk than the EW and MAD-Entropy. Moreover, the EW performance ratio is higher than MAD-Entropy and MAD. Nevertheless, it differs from the Sharpe ratio, where the MAD-Entropy Sharpe ratio is higher than the EW and MAD. In the 2018-2019 period, the EW has a higher mean return than MAD-Entropy and MAD. The EW also gets a higher Sharpe and performance ratio score than MAD-Entropy and MAD. The 2020-2021 period is unlike the portfolio of 3 and 5 stocks for the same period. The Equal Weight and MAD-Entropy methods have a positive mean return, while MAD has a negative mean return. Sharpe and Performance ratio of the MAD-Entropy is higher than the EW and MAD.

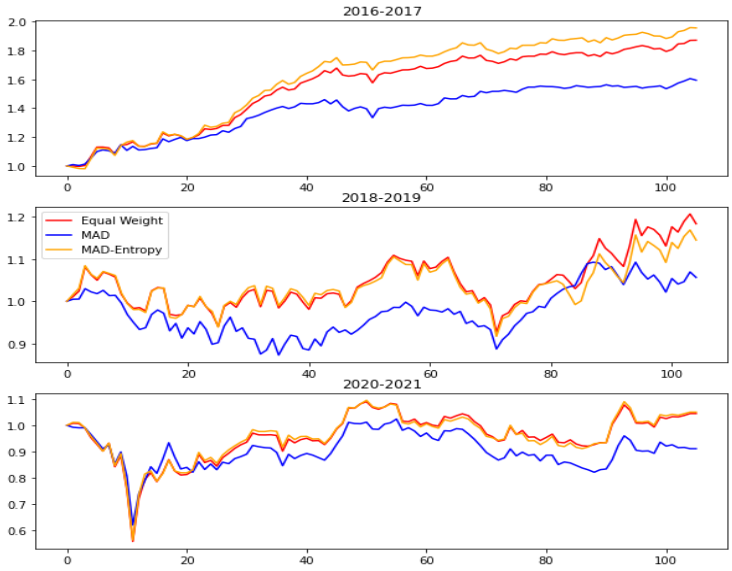

Fig. 4 Portfolio growth of 7 stocks

Fig. 4 shows the portfolio growth of 7 stocks. During the 2016-2017 period, the MAD-Entropy had the highest final portfolio value compared to the other two methods. In the 2018-2019 period, the EW produces the highest final return, slightly higher than the MAD-Entropy. In the 2020-2021 period, the Equal Weight and MAD-Entropy have a positive portfolio growth. It differs from MAD, which has a negative portfolio growth.

D. 26 stocks

Table VIII Statistics of 26 stocks

Method	Mean Return			Standard Deviation		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.00570	**0.0034**	0.0003	0.02364	0.0324	**0.0453**
Equal Weight	0.00731	0.0026	**0.0020**	**0.02297**	**0.0285**	0.0491
MAD-Entropy	**0.00912**	0.0027	0.0020	0.02482	0.0294	0.0482

Table IX Ratio of 26 stocks

Method	Sharpe Ratio			Performance Ratio		
	2016-2017	2018-2019	2020-2021	2016-2017	2018-2019	2020-2021
MAD	0.24126	**0.1043**	0.0066	0.31863	**0.1355**	0.0095
Equal Weight	0.31834	0.0924	0.0415	0.42113	0.1175	0.0610
MAD-Entropy	**0.36750**	0.0903	**0.0417**	**0.48736**	0.1168	**0.0617**

Table VIII and Table IX shows statistics and ratio for portfolio with 26 stocks. In the 2016-2017 period, MAD-Entropy (0.009) method has a higher mean return than the other two methods, Equal Weight (0.007) and MAD (0.006). For the Sharpe and performance ratio, MAD-Entropy gets the highest score with a value of 0.487. The EW and MAD have a value of 0.421 and 0.319, respectively. During the 2018-2019 period, in terms of mean return, MAD (0.0034) outperformed Equal Weight (0.0026) and MAD-Entropy (0.0026). For the Sharpe and performance ratio, MAD has the highest value with 0.13, higher than the EW and MAD-Entropy with 0.117 and 0.116, respectively. In the 2020-2021 period, Equal Weight (0.00203) in terms of mean return has the highest value compared to MAD-Entropy (0.00201) and MAD (0.00030). For the performance ratio, MAD-Entropy got a higher value of 0.617 than Equal Weight and MAD, which reached 0.0609 and 0.0095, respectively.

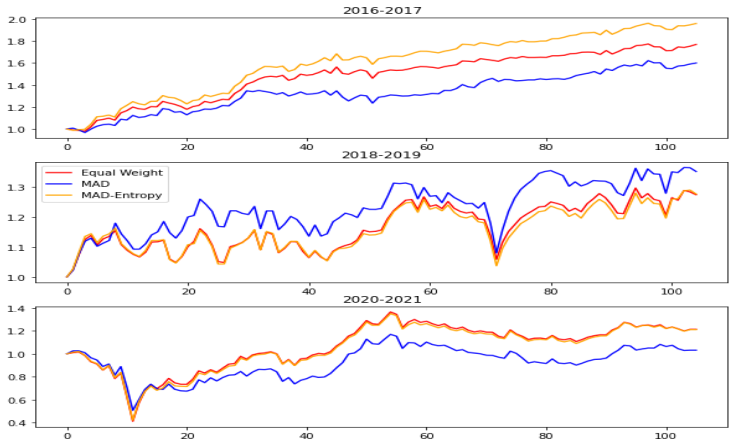

Fig. 5 Portfolio growth of 26 stocks

Fig. 5 shows portfolio growth for 26 stocks. In the 2016-2017 period, the MAD-Entropy method had the most significant portfolio growth compared to the other two methods. While in the 2018-2019 period, the MAD graph grows better than the Equal Weight and MAD-Entropy methods. During the 2020-2021 period, the MAD-Entropy and EW methods have almost similar growth values, while MAD produces a lower performance.

E. Comparison

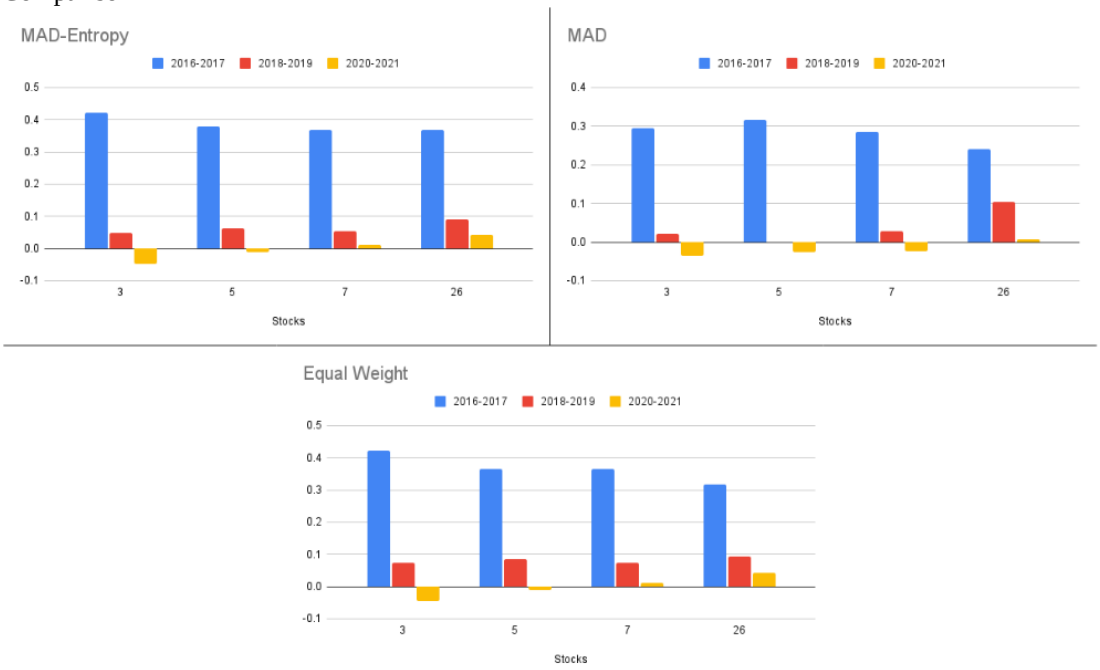

Fig. 6 Sharpe ratio from 3 methods on each scenarios

The relationship between portfolio performance (represented by the Sharpe ratio) with the number of stocks is shown in Figures 8, 9, and 10. In the 2016-2017 phase, where market conditions were good, the Sharpe ratio value decreased slightly along with the increase in stock for all methods. In the 2018-2019 phase, where market conditions are sideways, the number of stock increases does not increase the Sharpe ratio significantly, except for the MAD portfolio, where the Sharpe ratio value slightly increases. In the 2020-2021 phase, where market conditions experience turbulence due to the Covid-19 pandemic, the addition of the number of stocks causes an increase in the Sharpe ratio value, especially in MAD-Entropy and Equal Weight. Sharpe ratio value increases from negative to positive.

IV. CONCLUSION

MAD-Entropy method performed better than the other two methods for each scenario. For the 2018-2019 period, 3 out of 4 scenarios, the Equal Weight method is better, but the MAD method is better for the 26 stock scenario. For the last period, 2020-2021, the MAD-Entropy and Equal Weight methods have almost the same movement and ratio. For MAD in the 2020-2021 period, this is the worst performance of the MAD method due to the less diversity during the stock market crashes. The MAD shows a better performance for 26 stocks in the 2018-2019 period as the effect of the fantastic return of BRPT stocks is at 258.3%.

Overall, the portfolio performance of MAD-Entropy offers a higher portfolio return than the MAD. In contrast, the MAD portfolio has less risk than the MAD Entropy, indicated by less return standard deviation. However, a higher risk of the MAD-Entropy compensates much higher return, indicated by a higher Sharpe and Performance Ratio. This result shows that the entropy contributes to the better performance of the MAD. Moreover, the performance of the MAD-Entropy is sometimes slightly lower and sometimes higher than the

fully diversified portfolio, namely the Equal Weight portfolio. Nevertheless, their performance is not far distinctive.

During the turbulence phase due to the covid-19 pandemic, a portfolio with many stocks shows more stability, and the average portfolio return is still positive, especially for MAD-Entropy and Equal Weight portfolios.

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