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Multivariate Markov Chain Model for Sales Demand Estimation in a Company

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Abstract

Estimation of the number of demands for a product must be done correctly, so that the company can get maximum profit. Therefore, this study discusses how to estimate the amount of sales demand in a company correctly. The model that will be used to estimate sales demand is the Multivariate Markov Chain Model. This model can estimate the future state by observing the present state. The model requires parameter estimation values first, namely the transition probability matrix and the weighted Markov chain, where in previous studies an estimation of the transition probability matrix has been carried out, so that in this study we will continue to estimate the weighted Markov chain parameters. This model is compatible with 5 data sequences (product types) defined as product 1, product 2, product 3, product 4, and product 5, with 6 conditions (no sales volume, very slow-moving, slow-moving, standard, fast moving, and very fast moving). As the result, the state probability for product 1, product 2 and product 3 in company 1 are stationary at state 6 (very fast moving), product 4 and product 5 are stationary at state 2 (very slow moving).

Keywords: Multivariate Markov chain model, demand, estimation, the weighted Markov chain

I. Introduction

Markov chain was first created by a Russian professor named Andrei A. Markov (1856-1922), the following is Markov's statement which was later referred to as Markov property (Markovian Property): "The conditional probability of a future event, given that the future event is known. past and present events are independent of past events and dependent only on present events." [1] The formulation of the problem in this research is how to model the optimization problem of sales demand by using the multivariate Markov chain model. In previous studies [2], the estimation of the transition probability matrix has been carried out, so that in this research it will be continued to estimate the weighted Markov chain parameters. This model will be solved by linear program optimization method, using *linprog* facility in *Matlab* software.

II. Literature Review

In this multivariate Markov chain model, [3] it is assumed that there are s categories of categorical data (product types), each of which has m states (for example: many, few, etc.).

$$M = \{1, 2..., m\}$$

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Let $\boldsymbol{x}_n^{(j)}$ be the state probability vector of the *j*th sequence (product types) at time *n*. If at the *n*th time, the *j*th sequence (product types) is in state *l*, then it can be written [4]:

$$\boldsymbol{x}_n^{(j)} = \boldsymbol{e}_l = (0,\ldots,0,\underbrace{1,}_{\text{state } l},0,\ldots,0)^T.$$

In constructing the multivariate Markov chain model, the following equation is assumed [5]:

$$\boldsymbol{x}_{n+1}^{(j)} = \sum_{k=1}^{s} \lambda_{jk} \boldsymbol{P}^{(jk)} \boldsymbol{x}_n^{(k)}, \text{for } j = 1,2,\ldots,s \tag{1}$$

where $\lambda_{jk} \geq 0, 1 \leq j,k \leq s$,and $\sum_{k=1}^{s} \lambda_{jk} = 1$, for $j = 1,2,\ldots,s$

Thus, based on equation (1), the state probability distribution of the sequence (product type) *j* at time (*n* +1) depends on the states of the sequence (product type) *j* and *k* at time *n* [6]. Here λ_{jk} is the weighted Markov chain which includes the effect of the sequence state (product type) *k* to *j*. As $\mathbf{P}^{(jk)}$ is the probability of the sequence state (product type) *k* to *j*,[7] and $\boldsymbol{x}_n^{(k)}$is the probability of the sequence state (product type) *k* at time *n*. [8] The following is writing in matrix:

$$\boldsymbol{x}_{n+1} = \begin{pmatrix} \boldsymbol{x}_{n+1}^{(1)} \\ \boldsymbol{x}_{n+1}^{(2)} \\ \vdots \\ \boldsymbol{x}_{n+1}^{(s)} \end{pmatrix} = \begin{pmatrix} \lambda_{11}\boldsymbol{P}^{(11)} & \lambda_{12}\boldsymbol{P}^{(12)} & \ldots & \lambda_{1s}\boldsymbol{P}^{(1s)} \\ \lambda_{21}\boldsymbol{P}^{(21)} & \lambda_{22}\boldsymbol{P}^{(22)} & \ldots & \lambda_{2s}\boldsymbol{P}^{(2s)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{s1}\boldsymbol{P}^{(s1)} & \lambda_{s2}\boldsymbol{P}^{(s2)} & \ldots & \lambda_{ss}\boldsymbol{P}^{(ss)} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_n^{(1)} \\ \boldsymbol{x}_n^{(2)} \\ \vdots \\ \boldsymbol{x}_n^{(s)} \end{pmatrix}$$

$$\equiv \quad \mathbf{Q} \quad \mathbf{x}_n \tag{2}$$

Model (2) shows that there is only a positive correlation between product lines, so it is assumed λ_{jk} to have a non-negative value [9]. This illustrates that increasing the probability of each sequence (product) at time *n* can only increase the probability of the state at time *(n+1)*. While in actual conditions it is often found a decrease in circumstances, for example from a behavior state to not selling. Therefore, in order for this model to include a state derivation, [10] the state probability vector $\boldsymbol{x}_n^{(j)}$must be negatively correlated with the probability vector $\boldsymbol{z}_{n+1}^{(j)}$ as follows:

$$\boldsymbol{z}_{n+1}^{(j)} = \frac{1}{m-1}(\mathbf{1} - \boldsymbol{x}_n^{(j)}) \tag{3}$$

1 is a vector of one, and $\frac{1}{m-1}$ is a normalization constant with $m > 2$.

The next step is to build a new model for *s* sequences (products) $\boldsymbol{x}_n^{(1)}, \boldsymbol{x}_n^{(2)}, \ldots, \boldsymbol{x}_n^{(s)}$by including the probability of deriving the state as in equation (3).

The following is a new multivariate Markov chain model that includes state derivation:

$$\boldsymbol{x}_{n+1} = \begin{pmatrix} \boldsymbol{x}_{n+1}^{(1)} \\ \boldsymbol{x}_{n+1}^{(2)} \\ \vdots \\ \boldsymbol{x}_{n+1}^{(s)} \end{pmatrix} = \underbrace{\boldsymbol{\Lambda}^{+} \begin{pmatrix} \boldsymbol{x}_n^{(1)} \\ \boldsymbol{x}_n^{(2)} \\ \vdots \\ \boldsymbol{x}_n^{(s)} \end{pmatrix}}_{positive\ correlation} + \left(\frac{1}{m-1}\right) \underbrace{\boldsymbol{\Lambda}^{-} \begin{pmatrix} \mathbf{1} - \boldsymbol{x}_n^{(1)} \\ \mathbf{1} - \boldsymbol{x}_n^{(2)} \\ \vdots \\ \mathbf{1} - \boldsymbol{x}_n^{(s)} \end{pmatrix}}_{negative\ correlation} \tag{4}$$

where $\boldsymbol{\Lambda}^{+} = \begin{pmatrix} \lambda_{1,1}\boldsymbol{P}^{(11)} & \lambda_{1,2}\boldsymbol{P}^{(12)} & \ldots & \lambda_{1,s}\boldsymbol{P}^{(1s)} \\ \lambda_{2,1}\boldsymbol{P}^{(21)} & \lambda_{2,2}\boldsymbol{P}^{(22)} & \ldots & \lambda_{2,s}\boldsymbol{P}^{(2s)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{s,1}\boldsymbol{P}^{(s1)} & \lambda_{s,2}\boldsymbol{P}^{(s2)} & \ldots & \lambda_{s,s}\boldsymbol{P}^{(ss)} \end{pmatrix}$

and $\boldsymbol{\Lambda}^{-} = \begin{pmatrix} \lambda_{1,-1}\boldsymbol{P}^{(11)} & \lambda_{1,-2}\boldsymbol{P}^{(12)} & \dots & \lambda_{1,-s}\boldsymbol{P}^{(1s)} \\ \lambda_{2,-1}\boldsymbol{P}^{(21)} & \lambda_{2,-2}\boldsymbol{P}^{(22)} & \dots & \lambda_{2,-s}\boldsymbol{P}^{(2s)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{s,-1}\boldsymbol{P}^{(s1)} & \lambda_{s,-2}\boldsymbol{P}^{(s2)} & \dots & \lambda_{s,-s}\boldsymbol{P}^{(ss)} \end{pmatrix}$

where $\lambda_{jk} \geq 0, j = 1,2,\dots s, k = 1,2,\dots, s$, and $\sum_{k=-s}^{s} \lambda_{j,k} = 1$

Equation (4) is equivalent to,

$$\boldsymbol{x}_{n+1} = \begin{pmatrix} \boldsymbol{x}_{n+1}^{(1)} \\ \boldsymbol{x}_{n+1}^{(2)} \\ \vdots \\ \boldsymbol{x}_{n+1}^{(s)} \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{1,1} & \boldsymbol{H}_{1,2} & \dots & \boldsymbol{H}_{1,s} \\ \boldsymbol{H}_{2,1} & \boldsymbol{H}_{2,2} & \dots & \boldsymbol{H}_{2,s} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{s,1} & \boldsymbol{H}_{s,2} & \dots & \boldsymbol{H}_{s,s} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_{n}^{(1)} \\ \boldsymbol{x}_{n}^{(2)} \\ \vdots \\ \boldsymbol{x}_{n}^{(s)} \end{pmatrix} + \left(\frac{1}{m-1}\right) \begin{pmatrix} \boldsymbol{J}_{1,-1} & \boldsymbol{J}_{1,-2} & \dots & \boldsymbol{J}_{1,-s} \\ \boldsymbol{J}_{2,-1} & \boldsymbol{J}_{2,-2} & \dots & \boldsymbol{J}_{2,-s} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{J}_{s,-1} & \boldsymbol{J}_{s,-2} & \dots & \boldsymbol{J}_{s,-s} \end{pmatrix} \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \\ \vdots \\ \mathbf{1} \end{pmatrix}$$

$$\equiv \quad \mathbf{M}_s \quad \mathbf{x}_n + \mathbf{b}$$

where $\mathbf{H}_{j,k} = \left(\lambda_{j,k} - \frac{1}{m-1}\lambda_{j,-k}\right)\boldsymbol{P}^{(jk)}$, and $\mathbf{J}_{j,-k} = \lambda_{j,-k}\boldsymbol{P}^{(jk)}$
for $j = 1,2,..,s$ and $k = 1,2,..,s$

Therefore,

$$\begin{aligned} \boldsymbol{x}_{n+1} &= \boldsymbol{M}_s{}^1\boldsymbol{x}_n + \mathbf{Ib} \\ &= \mathbf{M}_s{}^2\boldsymbol{x}_{n-1} + (\mathbf{I} + \mathbf{M}_s{}^1)\boldsymbol{b} \\ &= \mathbf{M}_s{}^3\boldsymbol{x}_{n-2} + (\mathbf{I} + \mathbf{M}_s{}^1 + \boldsymbol{M}_s{}^2)\boldsymbol{b} \\ &\ \ \vdots \\ &= \mathbf{M}_s{}^{(n+1)}\boldsymbol{x}_{(n-n)} + \left(\boldsymbol{I} + \boldsymbol{M}_s{}^1 + \dots + \boldsymbol{M}_s{}^n\right)\boldsymbol{b} = \mathbf{M}_s{}^{(n+1)}\boldsymbol{x}_{(0)} + \left(\sum_{k=0}^{n} \boldsymbol{M}_s{}^k\right)\boldsymbol{b} \end{aligned}$$

where $\mathbf{M}_s{}^0 = \boldsymbol{I}$ dan $\mathbf{M}_s{}^n = \underbrace{\boldsymbol{M}_s.\boldsymbol{M}_s.\boldsymbol{M}_s.\dots.\boldsymbol{M}_s}_{\text{Multiplication of matrix } \mathbf{M}_s \text{ by } n \text{ times}}$

The model used has a stationary distribution for a certain norm matrix $\|.\|$ (in this case $\|\mathbf{M}_s\|_\infty$) where $\|\mathbf{M}_s\| < 1$. For any matrix **M** size *nxn* choose:

$$\|\mathbf{M}\|_\infty = max\left\{\sum_{j=1}^{n} \left|\mathbf{M}_{ij}\right|\right\}$$

In this case it can be written:

$$\lim_{n\to\infty} \boldsymbol{x}_n = \lim_{n\to\infty} \sum_{k=0}^{n} \boldsymbol{M}_s^k \boldsymbol{b} = (\boldsymbol{I} - \boldsymbol{M}_s)^{-1}\boldsymbol{b}$$

Furthermore,

$$\|\mathbf{M}_s\|_\infty \leq \max_{1\leq k\leq s}\left\{m\left|\lambda_{j,j} - \frac{1}{m-1}\lambda_{j,-j}\right| + \sum_{j\neq k}^{n}\left|\lambda_{j,k} - \frac{1}{m-1}\lambda_{j,-k}\right|\right\}$$

The smaller the $\|\mathbf{M}_s\|_\infty$, the faster the convergence rate [9].

III. Research Method

The presentation of the experimental methods should be clear and complete in every detail facilitating reproducibility by other scientists. The value $\lambda_{j,k}$ can be estimated by minimizing the difference between the left and right sides of equation (4). This minimization process can be solved by solving a linear program optimization problem for a certain norm matrix, namely: $\|.\|_1$ or $\|.\|_\infty$ [6]. The following is the meaning of the two matrix norms [11]:

Let matrix $\mathbf{A} = [a_{ij}] \epsilon\, \mathbb{C}^{nxn}$, then

1. $\|\mathbf{A}\|_1 = \max_j \sum_{i=1}^{n} |a_{ij}|$ (the largest number of column modulus)
2. $\|\mathbf{A}\|_\infty = \max_i \sum_{j=1}^{n} |a_{ij}|$ (the largest number of row modulus)

The writer chooses to use $\|.\|_\infty$ to estimate the value $\lambda_{j,k}$. The following is a linear program optimization problem for the norm matrix $\|.\|_\infty$:

$$\begin{cases} \min_{\lambda} \max_{i} \left| \left[\left(\sum_{k=1}^{s} \boldsymbol{M}_s(\lambda).\boldsymbol{x}_n{}^{(k)} + \boldsymbol{b} \right) - \boldsymbol{x}_n{}^{(j)} \right]_i \right| \\ \text{subject to} \\ \sum_{k=-s}^{s} \lambda_{j,k} = 1, \forall j = 1,2,\dots,s \\ \lambda_{j,k} \geq 0, \forall k \end{cases} \tag{5}$$

The optimization problem (5) is equivalent to:

$$\begin{cases} \min_{\lambda} \max_{i} \left| \left[\boldsymbol{a}^{(j)} - \boldsymbol{x}_n{}^{(j)} \right]_i \right| \\ \text{subject to} \\ \boldsymbol{a}^{(j)} = \sum_{k=1}^{s} \left(\underbrace{\left(\lambda_{j,k} - \frac{1}{m-1}\lambda_{j,-k} \right) \boldsymbol{P}^{(jk)}}_{\boldsymbol{M}_s} \boldsymbol{x}_n{}^{(k)} + \underbrace{\frac{1}{m-1}\lambda_{j,-k}\boldsymbol{P}^{(jk)}\mathbf{1}}_{\boldsymbol{b}} \right) \\ \sum_{k=-s}^{s} \lambda_{j,k} = 1, \forall j = 1,2,\dots,s \\ \lambda_{j,k} \geq 0, \forall j = 1,2,\dots s, k = \pm 1, \pm 2, \dots, \pm s \end{cases} \tag{6}$$

The Where 1 is a vector of size mx1 where each entry contains the number 1.

The linear programming optimization problem (6) can be formulated as a linear programming problem as follows:

$$\begin{cases} \min_{\lambda} w_j \\ \text{subject to} \\ \boldsymbol{w}_j \geq \boldsymbol{a}^{(j)} - \boldsymbol{x}_n{}^{(j)}, \\ \boldsymbol{w}_j \geq -\boldsymbol{a}^{(j)} + \boldsymbol{x}_n{}^{(j)}, \\ \boldsymbol{w}_j = \begin{pmatrix} w_j \\ w_j \\ \vdots \\ w_j \end{pmatrix}, w_j \text{ as much as m (condition)} \\ \boldsymbol{a}^{(j)} = \sum_{k=1}^{s} \left(\left(\lambda_{j,k} - \frac{1}{m-1}\lambda_{j,-k} \right) \boldsymbol{P}^{(jk)} \boldsymbol{x}^{(k)} + \frac{1}{m-1}\lambda_{j,-k}\boldsymbol{P}^{(jk)}\mathbf{1} \right) \\ w_j \geq 0, \\ \sum_{k=-s}^{s} \lambda_{j,k} = 1, \lambda_{jk} \geq 0, \forall k \end{cases} \tag{7}$$

The following is the optimization problem (7) can be solved by using the linprog facility in Matlab

IV. RESULTS AND DISCUSSION

A. The Current State Probability

The following is the current state probability value for five products in company 1 with each product having 6 states:

$$\boldsymbol{x}_n^{(1)} = \begin{pmatrix} 0.0818 \\ 0.4052 \\ 0.0483 \\ 0.0335 \\ 0.0037 \\ 0.4275 \end{pmatrix}, \boldsymbol{x}_n^{(2)} = \begin{pmatrix} 0.3680 \\ 0.1970 \\ 0.0335 \\ 0 \\ 0.0037 \\ 0.3978 \end{pmatrix}, \boldsymbol{x}_n^{(3)} = \begin{pmatrix} 0.1450 \\ 0.2045 \\ 0.0186 \\ 0 \\ 0.0037 \\ 0.6283 \end{pmatrix}, x_n^{(4)} = \begin{pmatrix} 0 \\ 0.3569 \\ 0.1338 \\ 0.1896 \\ 0.0632 \\ 0.2565 \end{pmatrix}, x_n^{(5)} = \begin{pmatrix} 0 \\ 0.3569 \\ 0.1227 \\ 0.2268 \\ 0.0520 \\ 0.2416 \end{pmatrix}$$

The greatest probability value for a product describes the state of the product. The current state probability in the fiveproducts form company 1: products 1, 2, and 3 are in the state of 6 (very fast moving), while products 4 and 5 are in state 2 (very slow moving).

B. The Weighted Markov Chain

After obtaining the transition probability matrix and the state probability vector in part A, the next step is to calculate the weighted Markov chain using the two results, according to the previous explanation in part III. The following are the weighted Markov chain for the five products in firm 3 with each product having 6 states:

$$\lambda_{j,k} = \begin{pmatrix} \lambda_{1,1} & \lambda_{1,2} & \lambda_{1,3} & \lambda_{1.4} & \lambda_{1,5} \\ \lambda_{2,1} & \lambda_{2,2} & \lambda_{2,3} & \lambda_{2.4} & \lambda_{2,5} \\ \lambda_{3,1} & \lambda_{3,2} & \lambda_{3,3} & \lambda_{3.4} & \lambda_{3,5} \\ \lambda_{4,1} & \lambda_{4,2} & \lambda_{4,3} & \lambda_{4.4} & \lambda_{4,5} \\ \lambda_{5,1} & \lambda_{5,2} & \lambda_{5,3} & \lambda_{5.4} & \lambda_{5,5} \end{pmatrix} = \begin{pmatrix} 0 & 0.9874 & 0 & 0 & 0 \\ 0 & 0.9806 & 0 & 0 & 0 \\ 0 & 0.9494 & 0 & 0 & 0.0289 \\ 0 & 0 & 0 & 0.0067 & 0.9933 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_{j,-k} = \begin{pmatrix} \lambda_{1,-1} & \lambda_{1,-2} & \lambda_{1,-3} & \lambda_{1,-4} & \lambda_{1,-5} \\ \lambda_{2,-1} & \lambda_{2,-2} & \lambda_{2,-3} & \lambda_{2,-4} & \lambda_{2,-5} \\ \lambda_{3,-1} & \lambda_{3,-2} & \lambda_{3,-3} & \lambda_{3,-4} & \lambda_{3,-5} \\ \lambda_{4,-1} & \lambda_{4,-2} & \lambda_{4,-3} & \lambda_{4,-4} & \lambda_{4,-5} \\ \lambda_{5,-1} & \lambda_{5,-2} & \lambda_{5,-3} & \lambda_{5,-4} & \lambda_{5,-5} \end{pmatrix} = \begin{pmatrix} 0.0113 & 0 & 0.0113 & 0 & 0 \\ 0.0194 & 0 & 0 & 0 & 0 \\ 0.0199 & 0.0018 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where $\sum_{k=1}^{5}(\lambda_{j,k} + \lambda_{j,-k}) = 1$,for each j, where $j = 1,2,..,5$

After getting the transition probability matrix $\boldsymbol{P}^{(jk)}$, the state probability vector $\boldsymbol{x}_n^{(s)}$, the probability weigth $\lambda_{j,k}$ and $\lambda_{j,-k}$, then the multivariate Markov chain model simulation can be performed to estimate the sales demand for five products in company 1 with each product having six conditions. This model simulation results in the form of a probability vector for the future, where the largest probability value in a row (state) is the state of the product.

C. The Future State Probability Vector

The following are the future state probability values for the five products in company 1, with each product having 6 states:

$$\boldsymbol{x}_{n+1}^{(1)} = \begin{pmatrix} 0.0819 \\ 0.4039 \\ 0.0490 \\ 0.0348 \\ 0.0039 \\ 0.4264 \end{pmatrix}, \boldsymbol{x}_{n+1}^{(2)} = \begin{pmatrix} 0.3662 \\ 0.1989 \\ 0.0336 \\ 0 \\ 0.0037 \\ 0.3977 \end{pmatrix}, \boldsymbol{x}_{n+1}^{(3)} = \begin{pmatrix} 0.1449 \\ 0.2048 \\ 0.0184 \\ 0 \\ 0.0037 \\ 0.6283 \end{pmatrix}, \boldsymbol{x}_{n+1}^{(4)} = \begin{pmatrix} 0 \\ 0.3586 \\ 0.1342 \\ 0.1896 \\ 0.0632 \\ 0.2543 \end{pmatrix}, \boldsymbol{x}_{n+1}^{(5)} = \begin{pmatrix} 0 \\ 0.3569 \\ 0.1227 \\ 0.2268 \\ 0.0520 \\ 0.2416 \end{pmatrix}$$

The greatest probability value for a product describes the state of the product. The future probability for the five products of company 1, namely: product 1, product 2, and product 3 are in state 6 (very fast moving), while product 4 and product 5 are in state 2 (very slow moving).

D. *Analysis*

The simulation results in sections A and C can be compared the results of the current state probability vector $\boldsymbol{x}_n$with the future state probability vector $\boldsymbol{x}_{n+1}$. As a result, the state probability at company 1 has a state that is not much different from the current state. So, it can be concluded that the the future state probability $\boldsymbol{x}_{n+1}$is stationary. If it is continued until $n+5$ it will be obtained a table of probabilities as follows:

TABLE I
TABLE OF THE STATE PROBABILITY IN COMPANY 1

	The State Probability																													
	Product 1						Product 2						Product 3						Product 4						Product 5					
t	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
n																														
n+1																														
n+2																														
n+3																														
n+4																														
n+5																														

Based on Table.1, the state probability for product 1, product 2 and product 3 in company 1 are stationary at state 6 (very fast moving), product 4 and product 5 are stnary at state 2 (very slow moving). So that we get a new conclusion that from the state probability $\boldsymbol{x}_{n+1}$until $\boldsymbol{x}_{n+5}$ are stationary (this strengthens the previous conclusion, namely the state probability $\boldsymbol{x}_{n+1}$ is stationary). Table 1 can only provide general information about the condition of a product. Therefore, to see in more detail the changes in the probability value of circumstances from time *t=n* to *t=n+5*, the author completes this study by presenting a bar graph of the probability value of each state in a company's product as follows:

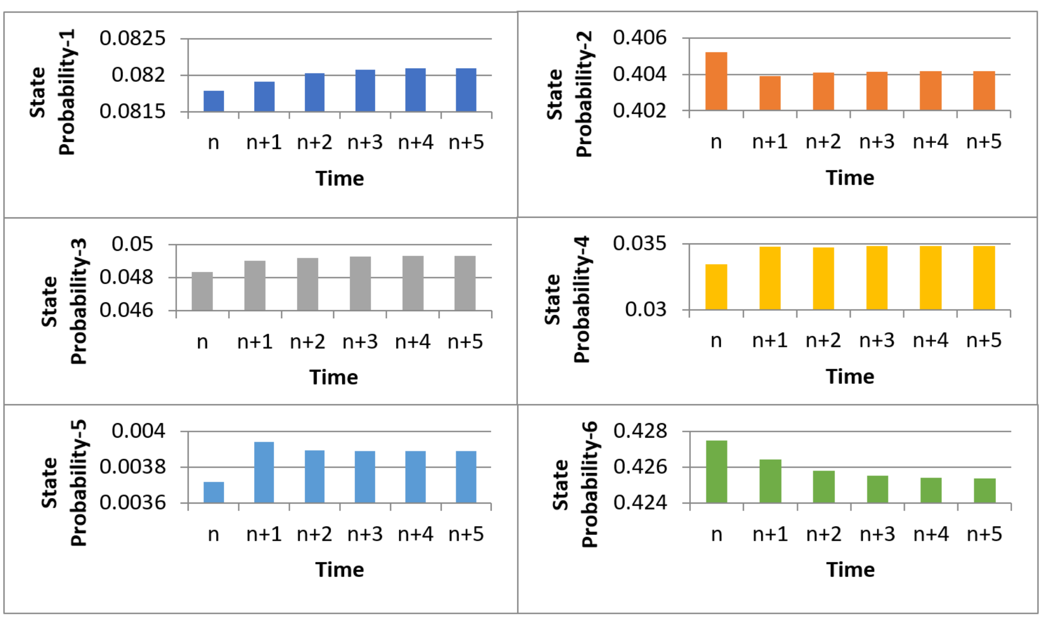

Fig. 1. The State Probability Graph of Each Product 1 in Company 1

Based on Fig. 1 it can be concluded that each state changes from time t=n to n+5, although the value of the change is very small. From the graph, it can also be seen that there is a trend, both uptrend and downtrend, such as state-1 (no sales volume) showing an uptrend, which means that within a certain period of time product 1 is likely to be in state 1. On the contrary, state-6 (very fast moving) showcing a downward trend, which means that within a certain period of time product 1 may no longer be in state-6, but has moved to state-1, for example. This information can be used by the company to determine the amount of production of a product in the future. In addition, this information can also be used in preparing the company's marketing strategy for the product.

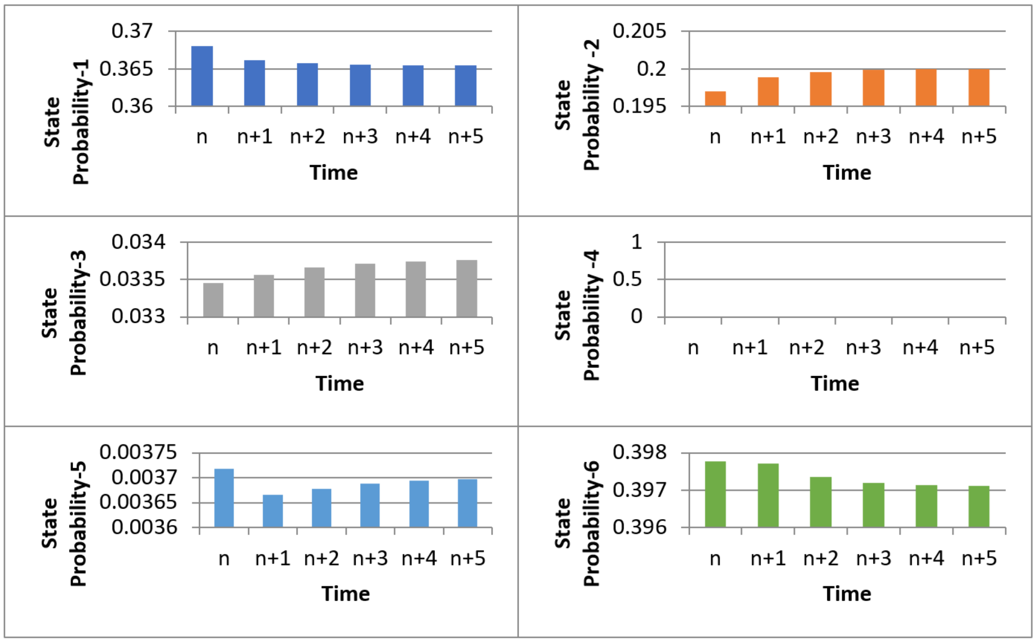

Fig. 2. The State Probability Graph of Each Product 2 in Company 1

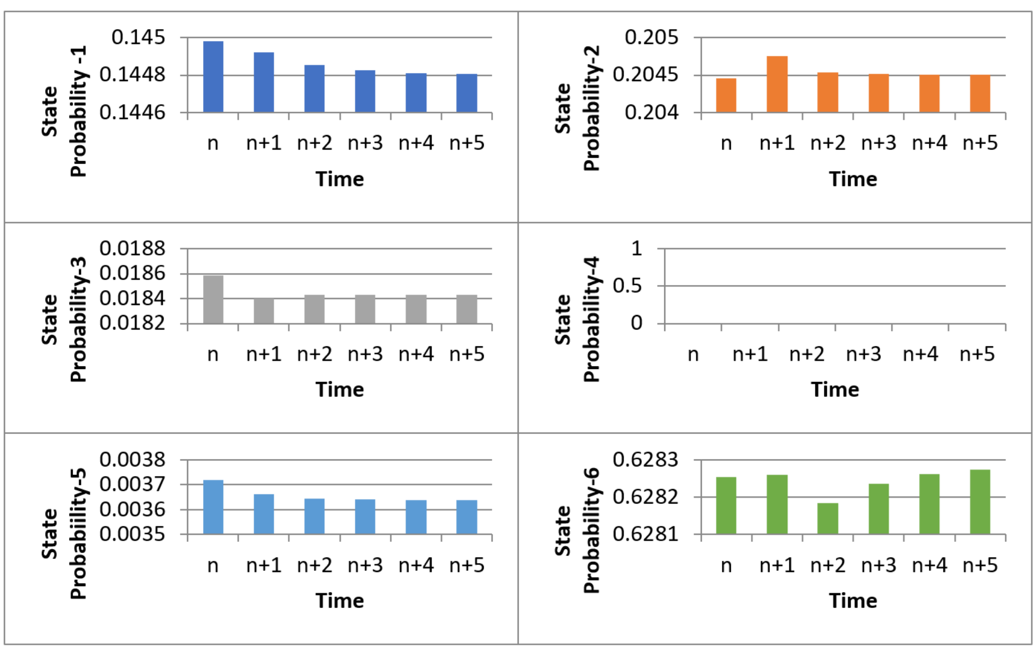

Fig. 3. The State Probability Graph of Each Product 3 in Company 1

Based on Fig.3 it can be seen that the uptrend is only found in state-6 (very fast moving) meaning that within a certain period of time product 3 will be in state-6 and will not move to another state.

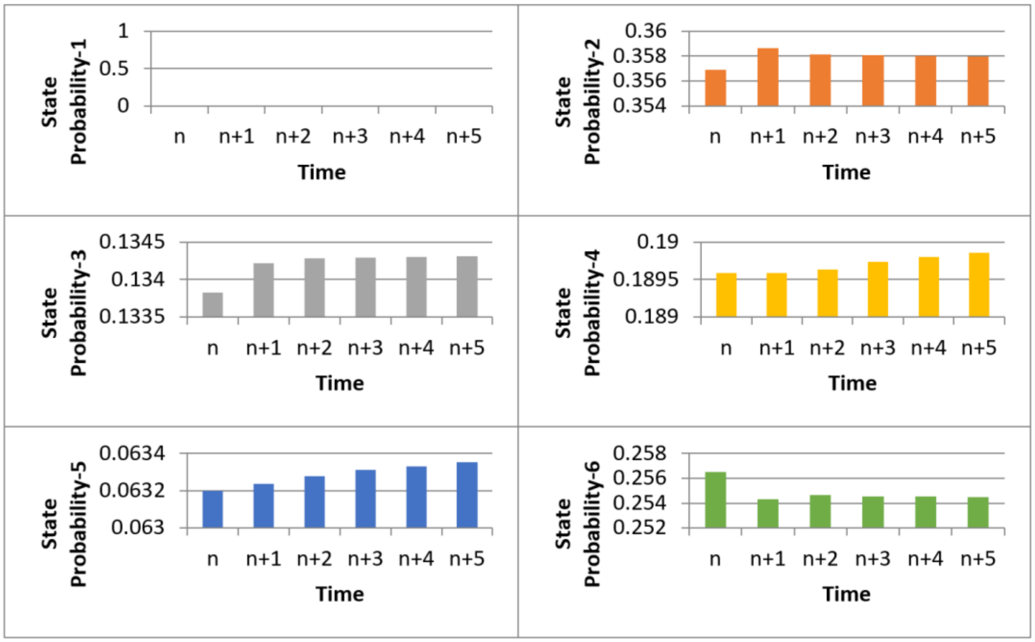

Fig. 4. The State Probability Graph of Each Product 4 in Company 1

Based on Fig.4 it can be seen that there is a trend, both an uptrend and a downtrend, such as state-3 (slow-moving), state-4 (standard) and state-5 (fast moving) experiencing an uptrend which means that within a certain period of time product 4 there is a possibility is in state-3, state-4, or state-5. On the other hand, state-2 (very slow-moving) experiences a downward trend, which means that within a certain period of time product 4 may no longer be in state-6, but has moved to state-3, state-4 or state-5.

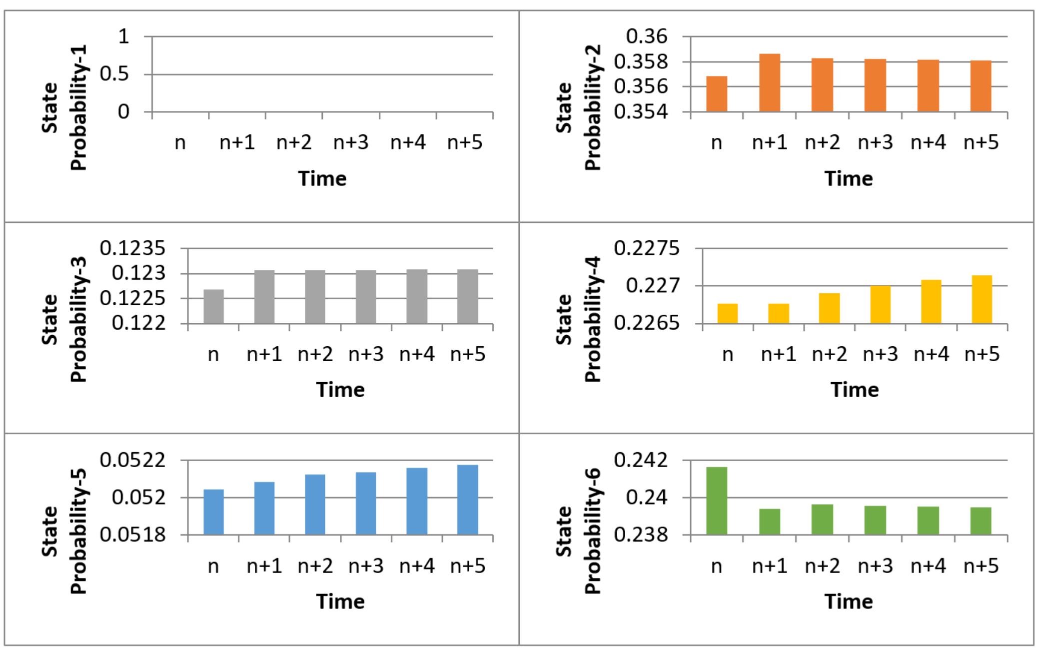

Fig. 5. The State Probability Graph of Each Product 5 in Company 1

Based on fig.5 it can be seen that there is a trend, both an uptrend and a downtrend, such as state-3 (slow-moving), state-4 (standard) and state-5 (fast moving) experiencing an uptrend which means that within a certain period of time product 4 there is a possibility is in state-3, state-4, or state-5. On the other hand, state-2 (very slow-moving) experiences a downward trend, which means that within a certain period of time product 4 may no longer be in state-6, but has moved to state-3, state-4 or state-5.

V. CONCLUSION

In completing the multivariate Markov chain model, it takes two parameters that must be estimated, namely: the transition probability matrix and the weighted Markov chain. The transition probability matrix has been calculated in previous studies. The weighted Markov chain is estimated by solving the linear program optimization problem using the *linprog* facility in Matlab. By substituting the transition probability matrix, the weighted Markov chain, and the state probability vector into the multivariate Markov chain model, the estimated value of future sales demand is obtained. The estimated value of future sales demand is a vector for the probability of the state of each product in the future. Where each row is the probability value for each state. If the probability value of a row has the greatest value, then the product in the future will be in that state. Based on the simulation results of the multivariate Markov chain model, the probability value of the future state x_{n+1} is stationary, that is x_n and $x_{n+1}$the value is not much different. If passed until x_{n+5} , the value is not much different from x_n . So, the probability value of the future state is not much different from the probability of the present state (stationary). When viewed in more detail using a bar graph, it can be seen that there is an uptrend or downtrend in the value of the state probability. This trend can describe the future state probability.

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